Answer the following problems and simplify your answers.

1. (36 pts) Evaluate the following integrals. **Show all work!**

   (a) $\int x^3 \ln x \, dx$ (b) $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} \, dx$ (c) $\int \frac{2x}{x^2-4x+3} \, dx$

2. (24 pts) With $I = \int_{-\pi/2}^{\pi/2} \cos x \, dx$, answer the following. **Leave your answer in exact form.**
   (a) Compute T_4 to estimate I.
   (b) Find a reasonable bound for the error $|E_T|$ of your calculation in part (a).
   (c) What is the smallest value of n that will guarantee the error $|E_T|$ is less than 10^{-5}?

3. (20 pts) Do the following integrals converge or diverge? Evaluate the convergent integrals.
   (a) $\int_0^{\infty} \frac{2e^{2x}}{1+e^{4x}} \, dx$
   (b) $\int_2^{\infty} \frac{1+\cos^2 x}{x-1} \, dx$

4. (20 pts) Let $\mathcal{R}$ be the region bounded by $y = 1 - x^2$ and $y = x - 1$.
   (a) Sketch and shade the region $\mathcal{R}$. Label all axes, curves, and intersection points.
   (b) Set up, **but do not evaluate**, integrals to determine each of the following:
      i. The area of $\mathcal{R}$ using integration with respect to x.
      ii. The area of $\mathcal{R}$ using integration with respect to y.

Trigonometric Identities

$$\cos^2(x) = \frac{1}{2}(1+\cos 2x) \quad \sin^2 x = \frac{1}{2}(1-\cos 2x) \quad \sin 2x = 2\sin x \cos x \quad \cos 2x = \cos^2 x - \sin^2 x$$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, u^2 < a^2$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C, u^2 > a^2$$

Midpoint Rule

$$\int_a^b f(x)\,dx \approx \Delta x[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)],\ \Delta x = \frac{b-a}{n},\ \overline{x}_i = \frac{x_{i-1}+x_i}{2},\ |E_M| \le \frac{K(b-a)^3}{24n^2}$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)],\ \Delta x = \frac{b-a}{n},\ |E_T| \le \frac{K(b-a)^3}{12n^2}$$