APPM 1360 Exam #3

Summer 2022: Carefully read the following important information.

- This exam is worth 100 points and has 5 questions.
- Write clearly, neatly and legibly, from left to right and top to bottom.
- Show **ALL** your work, simplifying and putting a box around your final answer.
- You must arrive at your answer through a logical, legible and understandable sequence of correct mathematical statements. Failure to do so will result in zero (0) points, regardless of whether or not you write the correct answer.
- Begin each problem on a new page.
- You are taking this exam in a proctored and honor code enforced environment. Thus, no calculators, cell phones (except for video of yourself in Zoom), or other electronic devices are permitted. Accessing any other resources (textbooks, notes, internet resources, fellow students, other humans, *etc.*) is strictly prohibited. This exam is being administered with the use of **PROCTORIO.**
- When finished, scan your exam into a single pdf file with problems in the order shown on the exam (page 1 is problem 1, page 2 is problem 2, *etc.*)
- You have the entire class period to complete the exam, scan your work into a single pdf file and upload that file to GRADESCOPE, indicating which page contains each problem. You have 10 minutes after the exam to submit your work to GRADESCOPE.
- Give yourself enough time to make sure your submission uploaded correctly, is readable and is COMPLETE. Unreadable exams will not be graded. We will not accept late missing portions of exams. Report technical problems to your proctor. Do not leave your Zoom meeting until given permission by the proctor.

Question	Points	Score
1	24	
2	30	
3	18	
4	12	
5	16	
Total	100	

1. Determine if the series converge or diverge. Be sure to fully justify your answer and state what test that you used.

   (a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3n-1}$

   (b) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{6^{k-1}}$

   (c) (8 points) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5n-2n^3}{6n^3+3}\right)^n$

2. Determine the interval of convergence and radius of convergence for the following power series.

   (a) (15 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$

   (b) (15 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n!}$

3. (a) (10 points) Start with the Maclauren Series for $\frac{1}{1-x}$ to find a power series representation for $\frac{1}{1+2x^2}$. Show all work.

   (b) (8 points) Use your answer from part (a) to find its interval of convergence.

4. (12 points) Find the first 4 terms $(c_0 + c_1 x + c_2 x^2 + c_3 x^3)$ of the Maclauren series for $\cos(x + \frac{\pi}{2})$

5. (a) (10 points) Evaluate $\displaystyle\int_0^{0.4} \ln(1+x)dx$ as an infinite series.

   (b) (6 points) Estimate the error if you use the first 2 terms of the series in 5 (a) to approximate the value of the definite integral.

Taylor Series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Taylor's Formula

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

Frequently Used Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \qquad R = 1$$