APPM 1360 Exam #2

Summer 2021: Carefully read the following important information.

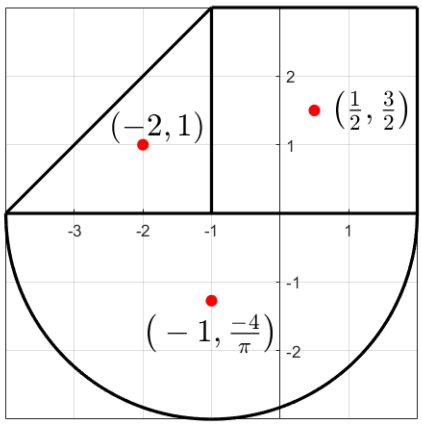
- This exam is worth 100 points and has 6 questions.
- Write clearly, neatly and legibly, from left to right and top to bottom.
- Show **ALL** your work, simplifying and putting a box around your final answer.
- You must arrive at your answer through a logical, legible and understandable sequence of correct mathematical statements. Failure to do so will result in zero (0) points, regardless of whether or not you write the correct answer.
- Begin each problem on a new page.
- You are taking this exam in a proctored and honor code enforced environment. Thus, no calculators, cell phones (except for video of yourself in Zoom), or other electronic devices are permitted. Accessing any other resources (textbooks, notes, internet resources, fellow students, other humans, *etc.*) is strictly prohibited. This exam is being administered with the use of **PROCTORIO.**
- When finished, scan your exam into a single pdf file with problems in the order shown on the exam (page 1 is problem 1, page 2 is problem 2, *etc.*)
- You have the entire class period to complete the exam, scan your work into a single pdf file and upload that file to GRADESCOPE, indicating which page contains each problem. You have 10 minutes after the exam to submit your work to GRADESCOPE.
- Give yourself enough time to make sure your submission uploaded correctly, is readable and is COMPLETE. Unreadable exams will not be graded. We will not accept late missing portions of exams. Report technical problems to your proctor. Do not leave your Zoom meeting until given permission by the proctor.

Question	Points	Score
1	20	
2	15	
3	15	
4	24	
5	6	
6	20	
Total	100	

1. Let $\mathscr{R}$ be the region bounded by the curve $y = \sqrt{x}$, the line $x = 4$, and the x-axis. The region $\mathscr{R}$ is rotated around the line $x = 6$ to form a solid.
   (a) (12 points) Set up an integral(s) for the volume of this solid using the Method of Cylindrical Shells. **EVALUATE THE INTEGRAL.**
   (b) (8 points) Set up an integral(s) for the volume of this solid using the Disk/Washer Method. **DO NOT EVALUATE THE INTEGRAL.**

2. (15 points) Consider the curve defined by $x = \frac{1}{2}y^2$ on $0 \le x \le 4$. Set up but **DO NOT EVALUATE** the surface area of the solid obtained by rotating the curve about the **x-axis**.

3. (15 points) Consider the region of uniform density $\rho = 1$ composed of a half circle, a square, and an isosceles triangle. Find the centroid for the region (the centroids of each smaller region are given).



4. For each of the following, determine whether the sequence converges or diverges. Explain your work in each case.
   (a) (6 points) $a_n = \ln(2n+1) - \ln(n)$
   (b) (6 points) $c_n = \cos(n\pi/4)$
   (c) (6 points) $a_n = \dfrac{2^n}{5n^2}$
   (d) (6 points) $b_n = \dfrac{3n+1}{(3n+1)!}$

5. (6 points) If $d_1 = 5$ and $d_{n+1} = 2d_n$, find a closed form expression for d_n and then determine whether the sequence converges or diverges.

6. (20 points) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2y}{x^2-1}$ given the initial condition $y(0) = 4$.

Useful Formulas

Trigonometric identities

$2\cos^2(x) = 1 + \cos(2x)$
$2\sin^2(x) = 1 - \cos(2x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin(u/a) + C, u^2 < a^2$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}|u/a| + C, u^2 > a^2$$

Center of Mass Integrals

$$m_{\text{total}} = \int_a^b \rho\left[f(x) - g(x)\right]\mathrm{d}x$$
$$M_y = \int_a^b \rho x\left[f(x) - g(x)\right]\mathrm{d}x$$
$$M_x = \int_a^b \frac{\rho}{2}\left[\left(f(x)\right)^2 - \left(g(x)\right)^2\right]\mathrm{d}x$$
$\bar{x} = \frac{M_y}{m_{\text{total}}}$ and $\bar{y} = \frac{M_x}{m_{\text{total}}}$