APPM 1360 Exam #1

Summer 2021: Carefully read the following important information.

- This exam is worth 100 points and has 4 questions.
- Write clearly, neatly and legibly, from left to right and top to bottom.
- Show **ALL** your work, simplifying and putting a box around your final answer.
- You must arrive at your answer through a logical, legible and understandable sequence of correct mathematical statements. Failure to do so will result in zero (0) points, regardless of whether or not you write the correct answer.
- Begin each problem on a new page.
- You are taking this exam in a proctored and honor code enforced environment. Thus, no calculators, cell phones (except for video of yourself in Zoom), or other electronic devices are permitted. Accessing any other resources (textbooks, notes, internet resources, fellow students, other humans, *etc.*) is strictly prohibited.
- When finished, scan your exam into a single pdf file with problems in the order shown on the exam (page 1 is problem 1, page 2 is problem 2, *etc.*)
- You have the entire class period to complete the exam, scan your work into a single pdf file and upload that file to GRADESCOPE, indicating which page contains each problem. You have 10 minutes after the exam to submit your work to GRADESCOPE.
- Give yourself enough time to make sure your submission uploaded correctly, is readable and is COMPLETE. Unreadable exams will not be graded. We will not accept late missing portions of exams. Report technical problems to your proctor. Do not leave your Zoom meeting until given permission by the proctor.

Question	Points	Score
1	24	
2	20	
3	20	
4	36	
Total	100	

1. (24 total points) Let $\mathscr{R}$ be the region in the first quadrant bounded by the curves $y = 2 + x^2$ on the right, $y = 5$ on top, and $y = 3x + 2$ on the left.
   (a) (5 points) Sketch the region. Be sure to label all axes, curves, and intersection points.
   (b) (7 points) Set up the dx integral(s) (integral(s) with respect to x) which, if evaluated, would give the area of the region. **DO NOT EVALUATE**.
   (c) (7 points) Set up the dy integral(s) (integral(s) with respect to y) which, if evaluated, would give the area of the region. **DO NOT EVALUATE**.
   (d) (5 points) Find the area of the region $\mathscr{R}$ from either (b) or (c).

2. (20 total points)
   (a) (10 points) Consider the curve $y = 2x^3$. Use the Trapezoid Rule with $n = 4$ subintervals to estimate the integral of this curve from $x = 1$ to $x = 4$.
   (b) (10 points) Estimate the error in the calculation from part (a)

3. (20 total points)
   (a) (10 points) Does the improper integral $\displaystyle\int_1^2 \frac{1}{\sqrt{x-1}}\,dx$ converge? If yes, to what?
   (b) (10 points) Does the improper integral $\displaystyle\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx$ converge? If yes, to what?

4. (36 total points) Evaluate the following integrals.
   (a) (12 points) $\displaystyle\int_1^4 \frac{1}{\sqrt{y}\,\left(2y - \sqrt{y}\right)}\,dy$
   (b) (12 points) $\displaystyle\int \tan^3 t\,dt$
   (c) (12 points) $\displaystyle\int \frac{dx}{x(x^2-1)^{3/2}}$ (Here we assume $x > 1$.)

Useful Formulas

Trigonometric identities

$2\cos^2(x) = 1 + \cos(2x)$
$2\sin^2(x) = 1 - \cos(2x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin(u/a) + C, u^2 < a^2$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}|u/a| + C, u^2 > a^2$$

Midpoint Rule

$\int_a^b f(x)dx \approx M_n = h\left[f(\bar{x_1}) + f(\bar{x_2}) + \cdots + f(\bar{x_n})\right]$, where $h = \frac{b-a}{n}$ and $\bar{x}_i = \frac{x_i + x_{i-1}}{2}$

$|E_M| \leq \frac{K(b-a)^3}{24n^2}$

Trapezoidal Rule

$\int_a^b f(x)dx \approx T_n = \frac{h}{2}\left[f(\bar{x_1}) + 2f(\bar{x_2}) + \cdots + 2f(\bar{x_{n-1}}) + f(\bar{x_n})\right]$, where $h = \frac{b-a}{n}$

$|E_T| \leq \frac{K(b-a)^3}{12n^2}$