1. [APPM 1360 Exam (30 pts)] Evaluate the following integrals.
   
   (a) \( \int \frac{x^2}{\sqrt{4-x^2}} \, dx \)  
   (b) \( \int_0^\infty xe^{-2x} \, dx \)

2. [APPM 1360 Exam (40 pts)] The following problems are not related.
   
   (a) Consider the sequence \( \{ -\frac{6}{5}, 1, -\frac{5}{6}, \frac{25}{36}, \ldots \} \).

   i. Does the sequence converge? If so, find its limit. If not, explain why not.

   ii. Does the series \(-\frac{6}{5} + 1 - \frac{5}{6} + \frac{25}{36} \cdots\) converge? If so, find its sum. If not explain why not.

   (b) The binomial series for \( \frac{1}{(2+x)^3} \) is \( \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n \).

   i. For what values of \( x \) does the series converge?

   ii. Find \( \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}3^n} \)

3. [APPM 1360 Exam (30 pts)] The following problems are not related.
   
   (a) (20 pts) Consider the curve \( C \) parameterized by \( x(t) = t^3, y(t) = t^2 \).

   i. Find the length of the curve for \( 0 \leq t \leq 2\sqrt{3}/3 \).

   ii. What is the slope of the tangent line to the curve at the point \( (x,y) = (8, 4) \)?

   (b) (10 pts) Consider the region between \( y = x^2 + 4 \) and \( y = 8 \) for \( 0 \leq x \leq 2 \). Set up, but do not evaluate, integral(s) that would give the volume of the solid obtained by rotating the region about the \( x \)-axis in the given way:

   i. As an integral with respect to \( x \).

   ii. As an integral with respect to \( y \).

4. [APPM 1360 Exam (20 pts)] Let \( f(x) = \sqrt{x} \).

   (a) Approximate \( f(x) \) by a Taylor polynomial of degree 3 centered at 1.

   (b) Use Taylor’s Formula to estimate the accuracy of the approximation for \( x \) in the interval \([0.9, 1.1]\). Your answer does not need to be simplified.

5. [APPM 1360 Exam (30 pts)] The following problems are not related.

   (a) Given the parametric equations \( x(\theta) = 5 \sin \theta, y(\theta) = 6 \cos \theta \), eliminate the parameter to determine the conic section described by the equations.

   (b) Sketch the polar curve \( r = -2 \sin \theta \) in the \( xy \)-plane. Label the intercepts using both polar and rectangular coordinates.

   (c) Find the area of the region outside \( r = 2 \) and inside \( r = 1 + 2 \cos \theta \). The region is shaded in the following figure.

\[ y \]
\[ x \]
\[ 2 \]
\[ 3 \]
Trigonometric Identities

\[ 2 \cos^2 x = 1 + \cos 2x \]
\[ 2 \sin^2 x = 1 - \cos 2x \]
\[ \sin 2x = 2 \sin x \cos x \]
\[ \cos 2x = \cos^2 x - \sin^2 x \]

Inverse Trigonometric Integral Identities

\[ \int \frac{du}{\sqrt{u^2 - a^2}} = \sin^{-1} \left( \frac{u}{a} \right) + C, \ u^2 < a^2; \]
\[ \int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \left( \frac{u}{a} \right) + C; \]
\[ \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left( \frac{u}{a} \right) + C, \ u^2 > a^2 \]

Taylor Stuff

Series: \[ \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n \]

Remainder Formula: \[ R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!} (x - a)^{n+1}, \ z \text{ strictly between } x \text{ and } a \]