1. [APPM 1360 Exam (16 pts)] Find the centroid of the lamina with constant density enclosed by the curves

\[ y = \sqrt{x}, \quad y = \sqrt{x} + 2, \quad x = 0, \quad x = 1 \]

2. [APPM 1360 Exam (20 pts)] Consider the curve \( y = \frac{\sqrt{3}}{4} \sqrt{16 - 4x^2} \) on the interval \(-2 \leq x \leq 2\). Find the area of the surface obtained by rotating the curve about the \( x \)-axis.

3. [APPM 1360 Exam (20 pts)] Find the solution of the differential equation that satisfies the given initial condition. Write your answer as an explicit function, that is, \( y = \cdots \).

\[
\frac{dy}{dt} = y(y + 1), \quad y(0) = 1
\]

4. [APPM 1360 Exam (24 pts)] Determine whether or not the following sequences converge or diverge. If the sequence converges, find its limit.

(a) \( \left\{ \sin \left( \frac{n\pi}{2} \right) \right\} \)
(b) \( \left\{ \frac{n^4}{\sqrt{n^3 - 3n^5 + 8}} \right\} \)
(c) \( a_n = \frac{(-1)^n \tan^{-1} n}{(n + 1)^{1/4}} \)

5. [APPM 1360 Exam (20 pts)] Consider the region bounded by the curves \( y = 2x \) and \( y = x^3 \), for \( 0 \leq y \leq 1 \). Set up, but do not evaluate, integrals to find the volume of the solid obtained by rotating the region about the line \( x = 2 \) using

(a) disks/washers
(b) cylindrical shells

FORMULAS ON NEXT PAGE
Trigonometric Identities

\[
\begin{align*}
2 \cos^2 x &= 1 + \cos 2x \\
2 \sin^2 x &= 1 - \cos 2x \\
\sin 2x &= 2 \sin x \cos x \\
\cos 2x &= \cos^2 x - \sin^2 x
\end{align*}
\]

Inverse Trigonometric Integral Identities

\[
\begin{align*}
\int \frac{du}{\sqrt{a^2 - u^2}} &= \sin^{-1} \left( \frac{u}{a} \right) + C, u^2 < a^2; \\
\int \frac{du}{a^2 + u^2} &= \frac{1}{a} \tan^{-1} \left( \frac{u}{a} \right) + C; \\
\int \frac{du}{u \sqrt{u^2 - a^2}} &= \frac{1}{a} \sec^{-1} \left( \frac{u}{a} \right) + C, u^2 > a^2
\end{align*}
\]

Center of Mass Integrals

\[
\begin{align*}
M &= \int_a^b \rho [f(x) - g(x)] \, dx \\
M_y &= \int_a^b \rho x [f(x) - g(x)] \, dx \\
M_z &= \int_a^b \frac{1}{2} \rho \left\{ [f(x)]^2 - [g(x)]^2 \right\} \, dx \\
\bar{x} &= \frac{M_y}{M} \quad \text{and} \quad \bar{y} = \frac{M_x}{M}
\end{align*}
\]