1. (25 pts) The following parts are not related.

(a) Evaluate the integral \( \int \frac{dx}{x^2\sqrt{1 + x^2}} \)

(b) Find an error bound for the Trapezoid Rule approximation to \( \int_{-3}^{-1} \frac{1}{2x^2} \, dx \) when \( n = 2 \).

2. (25 pts) The following parts are not related.

(a) Find the general solution to the differential equation \( \frac{dy}{dt} = -3yt^2 \).

(b) Consider a thin metal plate of uniform density defined by the region bounded by \( y = \sqrt{\ln x} \) and \( y = \sqrt{2x} \) for \( 1 \leq x \leq e \). If the moment about the \( x \)-axis is \( M_x = 3e^2 - 6 \) find the density of the plate.

3. (50 pts) The following parts are not related. Cite any theorems that you use.

(a) Consider the function \( f(x) = 2^x \).
   i. Find the Taylor series for \( f(x) \) centered at \( \alpha = 1 \). (Hint: \( \frac{d}{dx} 2^x = \ln 2 \cdot 2^x \)).
   ii. What can you conclude about the series’ radius of convergence if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 \)?
   iii. Find the exact error of the \( T_3 \) approximation to \( f(1) \).
   iv. Find an error bound for the \( T_3 \) approximation to \( 2^{1/2} \).

(b) Answer the following with “true” or “false.” If the statement is true justify by citing a theorem. If the statement is false give an example demonstrating it does not always hold.
   i. If \( b_n \geq 0 \) for every \( n \) and \( \lim_{n \to \infty} b_n = 0 \) then \( \{ (-1)^n b_n \} \) must converge to 0.
   ii. If \( \sum a_n \) and \( \sum b_n \) are divergent then \( \sum (a_n \cdot b_n) \) must also diverge.
   iii. If \( \{a_n\} \) converges then \( \sum a_n \) converges.
   iv. If \( \sum a_n = 3 \) then \( \lim_{n \to \infty} a_n = 0 \).
4. (50 pts) The following parts are not related.

(a) Consider the curve defined by the parametric equation

\[
C = \begin{cases} 
  x(t) = \sin t \\  y(t) = \cos^2 t 
\end{cases} \quad 0 \leq t \leq 2\pi
\]

i. Find the Cartesian representation of \( C \) by eliminating the parameter \( t \).
ii. Sketch the parametric curve \( C \) for \( 0 \leq t \leq 2\pi \).
iii. Use a plot of \( x(t) \) against \( t \) to discuss the directions of motion that a particle would travel on \( C \) as the parameter \( t \) increases.
iv. Use the parametric formula to set-up the integral to compute the area bounded by \( C \) and the \( x \)-axis.
v. Set-up an integral to compute the volume of the solid generated by rotating \( C \) about the \( y \)-axis.

(b) Consider the function \( r = 1 + 2 \sin \theta \). Sketch the curve on \( 0 \leq \theta \leq 2\pi \).

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**Trigonometric identities**

\[
\begin{align*}
\sin(2x) &= 2 \sin(x) \cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin^2(x) &= \frac{1}{2} (1 - \cos(2x)) \\
\cos^2(x) &= \frac{1}{2} (1 + \cos(2x))
\end{align*}
\]

**Inverse Trigonometric Integral Identities**

\[
\begin{align*}
\int \frac{du}{\sqrt{a^2 - u^2}} &= \sin^{-1}(u/a) + C \\
\int \frac{du}{a^2 + u^2} &= \frac{1}{a} \tan^{-1}(u/a) + C \\
\int \frac{du}{u\sqrt{u^2 - a^2}} &= \frac{1}{a} \sec^{-1}(u/a) + C
\end{align*}
\]

**Center of Mass Integrals**

\[
\bar{x} = \frac{M_y}{m_{\text{tot}}} \quad \text{and} \quad \bar{y} = \frac{M_x}{m_{\text{tot}}}
\]

\[
M_x = \int_a^b \frac{\rho}{2} \left[ (f(x))^2 - (g(x))^2 \right] dx \quad M_y = \int_a^b \rho x [f(x) - g(x)] dx
\]

**Integral Approximations**

**Midpoint:** \[
\int_a^b f(x)dx \approx \Delta x \left[ f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*) \right], \quad x_i^* = \frac{1}{2}(x_{i-1} + x_i), \quad |E_M| \leq \frac{K(b - a)^3}{24n^2}
\]

**Trapezoidal:** \[
\int_a^b f(x)dx \approx \frac{\Delta x}{2} \left[ f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right], \quad |E_T| \leq \frac{K(b - a)^3}{12n^2}
\]