1. (30 pts) Determine whether the following series converge absolutely, converge conditionally, or diverge. Be sure to cite any theorems that you use.

(a) \( \sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n^3} \)  
(b) \( \sum_{n=0}^{\infty} \frac{(3n)!}{(n!)^3} \)  
(c) \( \sum_{n=2}^{\infty} \frac{n}{n^{5/2} - 1} \)

2. (25) Short Answer: the following parts are not related.

(a) Suppose \( S_n = 1 - \frac{1}{(n + 1)!} \) is the \( n \)-th partial sum of a series \( \sum_{n=1}^{\infty} a_n \).

i. Does \( \sum_{n=1}^{\infty} a_n \) converge or diverge? If it converges, what does it converge to?

ii. Find an expression for \( a_n \). Simplify as much as possible.

(b) Suppose \( \sum_{n=0}^{\infty} c_n (x + 2)^n \) converges at \( x = 0 \) and diverges at \( x = -5 \).

i. True or False? The series converges at \( x = 2 \). Explain briefly.

ii. True or False? The series converges at \( x = -3 \). Explain briefly.

(c) Suppose that \( \{b_n\} \) is a sequence of positive numbers that converges to 3. Is the following series absolutely convergent, conditionally convergent, or divergent?

\( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \pi^{n/2}}{b_1 b_2 b_3 \cdots b_n} \)

(d) True or False? \( \sum_{n=1}^{\infty} \frac{1}{n^{1/n}} \) is convergent. Briefly justify your answer.

TURN OVER—More problems on the back!
3. (20 pts) Refer to the series below for the following questions. Be sure to cite any theorems that you use.

\[ \sum_{n=2}^{\infty} (-1)^n \frac{1}{4n - 5} \]

(a) Is the series absolutely convergent, conditionally convergent, or divergent?
(b) How many terms should be used in the \( S_n \) approximation to ensure the error satisfies \( |R_n| < 10^{-2} \)? Justify your answer.

4. (25 pts) Consider the function \( f(x) = \frac{\arctan(x)}{x} \).

(a) Find a power series for \( F(x) = \frac{1}{1 + x^2} \).
(b) Use the result of part (a) to find a power series for \( f(x) \). (Hint: \( \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2} \).)
(c) Find the radius and interval of convergence for the series in (b).
(d) Without further calculation give the radius of convergence for the power series of \( \int f(x) \, dx \). Briefly justify your answer.

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**Trigonometric identities**

\[
\begin{align*}
\sin(2x) &= 2 \sin(x) \cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin^2(x) &= \frac{1}{2} (1 - \cos(2x)) \\
\cos^2(x) &= \frac{1}{2} (1 + \cos(2x))
\end{align*}
\]

**Inverse Trigonometric Integral Identities**

\[
\begin{align*}
\int \frac{du}{\sqrt{a^2 - u^2}} &= \sin^{-1}(u/a) + C \\
\int \frac{du}{a^2 + u^2} &= \frac{1}{a} \tan^{-1}(u/a) + C \\
\int \frac{du}{u\sqrt{u^2 - a^2}} &= \frac{1}{a} \sec^{-1}(u/a) + C
\end{align*}
\]

**Geometric Power Series**

\[
\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n
\]

**Useful Limits: (\( x \) is fixed)**

\[
\begin{align*}
\lim_{n \to \infty} \sqrt[n]{n} &= 1 \\
\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n &= e^x \text{ for any } x \\
\lim_{n \to \infty} x^{\frac{1}{n}} &= 1 \text{ for } x > 0 \\
\lim_{n \to \infty} \frac{1}{n!} &= 0
\end{align*}
\]