On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 5 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.

- Show all work, simplify your answers Answers with no justification will receive no points. Please begin each problem on a new page.

- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or electronic devices are permitted.

1. (21 pts) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

   (a) \( \sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n} \)

   (b) \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} \)

   (c) \( \sum_{n=1}^{\infty} ne^{-n} \)

2. (18 pts) Consider the sequences \( \{a_n\}_{n=1}^{\infty} \) and \( \{b_n\}_{n=1}^{\infty} \) and that \( \sum_{n=1}^{\infty} a_n \) diverges and \( \sum_{n=1}^{\infty} b_n \) converges.

   No justification is necessary for the following questions. If the answer cannot be determined, write Indeterminate. If there are no values that satisfy the conditions, write None.

   (a) Find all real values of \( \alpha \) such that \( \sum_{n=1}^{\infty} \alpha \cdot a_n \) diverges.

   (b) Find all real values of \( \beta \) such that \( \sum_{n=1}^{\infty} (\beta + b_n) \) converges.

   (c) Find all real values of \( \gamma \) such that \( \sum_{n=1}^{\infty} (a_n + b_n) \) converges.

3. (21 pts) Complete the following parts in order:

   (a) Find a power series representation of \( \frac{-1}{(1+x)^2} \).

   (b) Find a power series representation of \( \frac{2}{(1+x)^3} \).

   (c) Find the sum \( \sum_{n=2}^{\infty} \frac{n(n-1)}{2^{n-2}} \) using your previous answers.
4. (20 pts) Show that
\[ \sum_{n=1}^{\infty} \left( -1 \right)^{n+1} \frac{n+1}{2^n} \]
converges. How many terms are required to approximate the sum to within an error of \( \frac{1}{4} \)?

5. (20 pts) Find the Interval of Convergence of the series
\[ \sum_{n=1}^{\infty} c_n x^n = 1 + 3x + x^2 + 3x^3 + x^4 + \cdots \]
where \( c_{2n} = 1 \) and \( c_{2n+1} = 3 \).

Useful Formulas

**Trigonometric identities**
\[
\begin{align*}
2 \cos^2(x) &= 1 + \cos(2x) \\
2 \sin^2(x) &= 1 - \cos(2x) \\
\sin(2x) &= 2 \sin(x) \cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x)
\end{align*}
\]

**Geometric Series**
\[
\frac{a}{1-r} = \sum_{n=0}^{\infty} ar^n \text{ when } |r| < 1
\]