On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 4 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.

- **Show all work, simplify your answers** Answers with no justification will receive no points. **Please begin each problem on a new page.**

- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or electronic devices are permitted.

1. (30 pts) Evaluate the following integrals.
   
   (a) \[ \int e^x \cos(2x) \, dx \]
   
   (b) \[ \int_0^2 \frac{12}{\sqrt{\pi}} \left( t^2 - 1 \right)^{3/2} \, dt \]
   
   (c) \[ \int \frac{x}{x^4 - x^2 - 2} \, dx \]

2. (25 pts) Consider the area bounded above by \( x = y^2 \) and below by \( x = y + 2 \) and the x-axis in the upper half plane.
   
   (a) Sketch the region. Be sure to label all axes, curves, and intersection points.
   
   (b) Set up the \( dx \) integral(s) (integral(s) with respect to \( x \)) which, if evaluated, would give the area of the region.
   
   (c) Set up the \( dy \) integral(s) (integral(s) with respect to \( y \)) which, if evaluated, would give the area of the region.
   
   (d) Evaluate the integral(s) from either (b) or (c).

3. (20 pts) Determine if the following integrals converge or diverge. For (a) determine the value to which the integral converges if the integral converges.
   
   (a) \[ \int_{-\infty}^{\infty} xe^{-x^2} \, dx \]
   
   (b) \[ \int_1^\infty \frac{1}{e^x - 2x} \, dx \]

4. (25 pts) Consider the function, \( f(x) = xe^x \)
   
   (a) Approximate the area under \( f(x) \) from \( x = 0 \) to \( x = 1 \) using Trapezoidal Rule with \( n = 4 \)
   
   (b) Estimate the error in the calculation from part (a).
Useful Formulas

**Trigonometric identities**

\[ 2 \cos^2(x) = 1 + \cos(2x) \]
\[ 2 \sin^2(x) = 1 - \cos(2x) \]
\[ \sin(2x) = 2 \sin(x) \cos(x) \]
\[ \cos(2x) = \cos^2(x) - \sin^2(x) \]

**Inverse Trigonometric Integral Identities**

\[
\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C, \quad u^2 < a^2
\]
\[
\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C
\]
\[
\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}|u/a| + C, \quad u^2 > a^2
\]

**Midpoint Rule**

\[
\int_a^b f(x) \, dx \approx M_n = h \left[ f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n) \right], \text{ where } h = \frac{b-a}{n} \text{ and } \bar{x}_i = \frac{x_i + x_{i-1}}{2}
\]
\[
|E_M| \leq \frac{K(b-a)^3}{24n^2}
\]

**Trapezoidal Rule**

\[
\int_a^b f(x) \, dx \approx T_n = \frac{h}{2} \left[ f(\bar{x}_1) + 2f(\bar{x}_2) + \cdots + 2f(\bar{x}_{n-1}) + f(\bar{x}_n) \right], \text{ where } h = \frac{b-a}{n}
\]
\[
|E_T| \leq \frac{K(b-a)^3}{12n^2}
\]