1. (16 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. For this problem, and all subsequent problems, explain your work and name any test or theorem that you use.

   (a) $\displaystyle\sum_{n=2}^{\infty}\left(-\frac{1}{2}\right)^n n^3$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n+2}{\sqrt{n^3+5}}$

2. (12 points) Use the Maclaurin series for $\ln(1+x)$ and $\ln(1-x)$ to find the Maclaurin series for $\ln\left(\dfrac{1+x}{1-x}\right)$. Write your answer using sigma notation and include the radius of convergence.
   (*Hint:* Write out the first few terms of the $\ln(1+x)$ and $\ln(1-x)$ series.)

3. (18 points)

   (a) Find a series representation for $\displaystyle\int_0^1 e^{-x^3}\,dx$.

   (b) Use the Alternating Series Estimation Theorem to approximate the value of the definite integral from part (a) with an error less than $1/20$. Fully simplify your answer. (You may assume that the hypotheses of the Alternating Series Estimation Theorem are satisfied.)

4. (28 points) Define a function $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}x^n$.

   (a) Determine the values of $x$ for which the series is absolutely convergent.

   (b) Find a Taylor series for $f'(x)$.

   (c) Find a closed form (non-series) expression for $xf'(-x)$.

5. (26 points) The following problems are not related.

   (a) Find $T_2(x)$, the second order Taylor polynomial, centered at $\pi/4$, for $f(x)=\sin(x)$.

   (b) Write the series in sigma notation and find its sum.

   $$\frac{1}{1!\,3}+\frac{1}{2!\,9}+\frac{1}{3!\,27}+\frac{1}{4!\,81}+\cdots$$

   (c) Consider the parametric equations given by $x=1+\cos t$ and $y=t+\pi$ for $-\pi\le t\le\pi$. Eliminate the parameter and sketch the curve. Indicate with an arrow the direction in which the curve is traced as $t$ increases.