1. (24 points) Evaluate each of the following:

$$(a) \quad \int \cos^5 x \, dx \qquad\qquad (b) \quad \int_1^{\infty} \frac{\arctan(1/x)}{x^2} \, dx$$

2. (28 points) Consider the integral $\displaystyle\int_0^1 e^{-x^2} \, dx$. All parts of this problem refer to this integral.

   (a) Write out an approximation for this integral using the midpoint rule with $n = 4$. (You DO NOT need to simplify your final answer. Your answer should be in a form that *could* be directly input into a calculator.)

   (b) Using the techniques from this course, what is the error bound of the approximation from (a)?

   (c) Use the MacLaurin series for e^x to find a series representation for this integral.

   (d) Use the Alternating Series Estimation Theorem to determine the error bound if the first four nonzero terms of the series from (c) are used to approximate the value of the integral. (Be sure to verify the conditions of the Alternating Series Estimation Theorem hold true in this example.)

3. (24 points) A gloopy is an intelligent species that lives on the planet Blorpy. Gloopies enjoy getting around the planet Blorpy with jet packs. This one particular gloopy takes off at $t = 0$, travels in the air above a straight street, and then lands back down on the street. Specifically, after t seconds, they are $x(t) = t^2 + 10t$ feet down the street with a height of $y(t) = 100t - 4t^2$ feet.

   (a) At what time does the gloopy land back on the street?

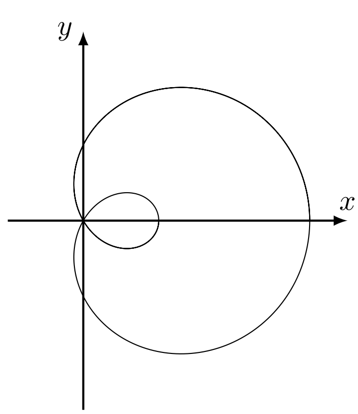
   (b) Set up but do not evalaute an integral for the length of the path that the gloopy traveled through the air between $t = 0$ seconds and the time found in (a).

   (c) Set up but do not evalaute an integral for the area between the gloopy's path and the street between $t = 0$ and the time found in (a).

4. (14 points) Find the sum of $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$.

5. (24 points) Consider the curve given by the following Polar equation which is plotted below:

$$r = 1 + 2\cos\theta, \qquad 0 \le \theta \le 2\pi$$



(a) Find the equation of the tangent line when $\theta = \frac{\pi}{3}$.

(b) Set up but do not evaluate an integral that equals the length of the inner loop.

6. (8 points) Consider the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$. Set up but do not evaluate an integral expression for the volume of the solid when this region is rotated about the line $y = -1$.

7. (8 points) Consider the following conic section:

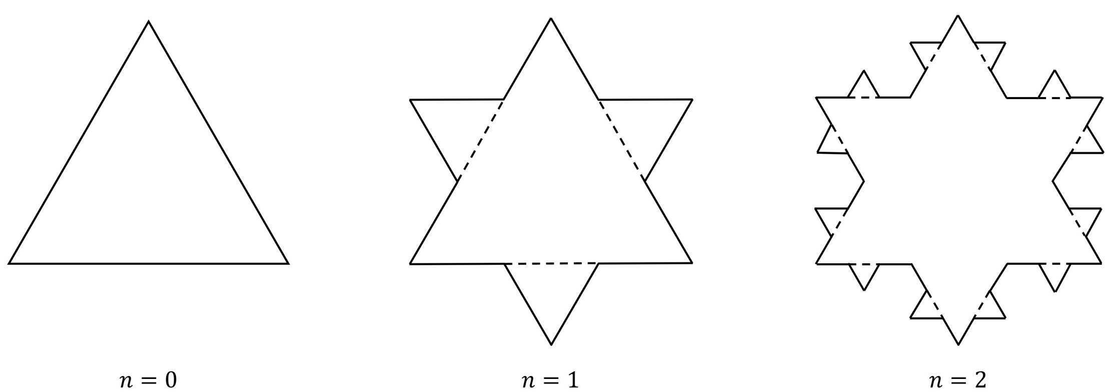
$$x^2 + 2y^2 - 6x + 5 = 0.$$

(a) Determine if this conic section is a **parabola**, an **ellipse**, or a **hyperbola**.

(b) If it is an ellipse or a hyperbola, determine its center. If it is a parabola, determine its vertex.

8. (20 points) Consider the fractal that is constructed iteratively as follows:

Begin with an equilateral triangle whose side lengths all equal 1. Each iteration of the construction process involves dividing each line segment from the previous iteration's perimeter into thirds. A new, smaller equilateral triangle is then built on the middle third of each such line segment, thereby increasing the area of the shape and the number of line segments comprising the perimeter. The initial triangle and the results of the first two iterations are depicted here:



(a) Let the sequence $\{a_n\}_{n=0}^{\infty}$ represent the total number of line segments on the perimeter of the shape after n iterations. Find a_n. (You should be counting the solid line segments, not the dashed line segments.)

(b) Let the sequence $\{b_n\}_{n=0}^{\infty}$ represent the length of each line segment on the perimeter of the shape after n iterations. Find b_n.

(c) Let the sequence $\{c_n\}_{n=1}^{\infty}$ represent the number of new triangles that are added to the shape during the n^{th} iteration. Find c_n.

(d) If the construction process is continued indefinitely, what would be the limiting area of the shape?
(*Note:* The area of an equilateral triangle with side length L is $\sqrt{3}\,L^2/4$.)