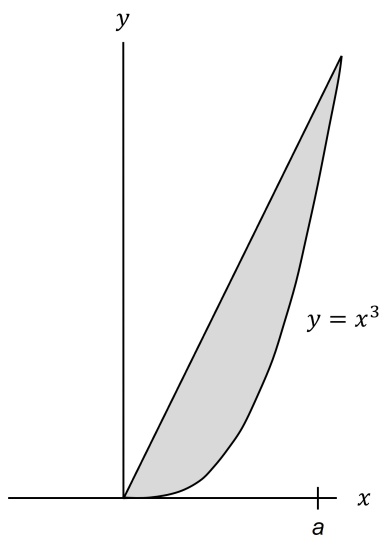
1. (10 points) Solve the following initial value problem:

$$\frac{dy}{dx} = x^2 \csc(y), \qquad y(2) = 0.$$

Write your answer in the form $y = f(x)$.

2. (12 pts) Consider the lamina depicted below, which is bounded above by a line through the origin and below by the curve $y = x^3$ on the interval $0 \le x \le a$. The line and the curve intersect at $x = 0$ and at $x = a$. The lamina has a uniform density of ρ. What value of a is needed so that $\bar{x} = 1$?



3. (28 pts) Consider the region $\mathcal{R}$, in quadrant I, bounded by the x-axis, the y-axis, $y = 2$, and $y = \ln(2x)$.

   (a) Use the grid below to sketch and shade the region $\mathcal{R}$. Label the coordinates of the intersections of two curves. (You may find it helpful to know that $e^2 \approx 7.4$.)

   (b) Set up but do not evaluate expressions involving integrals to determine each of the following:

   I. The volume of revolution found by revolving the given region about the y-axis using cylindrical shells.

   II. The area of the surface generated by rotating the curve $f(x) = \ln(2x)$ with $0 \le y \le 2$ about the y-axis.

   III. The perimeter of $\mathcal{R}$. (That is, find the arc length of the entire perimeter of $\mathcal{R}$.)

4. (27 pts) Determine if each of the following converges or diverges. Be sure to fully justify your answers using the techniques learned in this course. If you use a Test or Theorem, be sure to state its name and show its hypotheses are satisfied.

   (a) $\left\{ \dfrac{5(2n+1)! - n!}{(2n+1)!} \right\}_{n=1}^{\infty}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (n+2)e^{-n}$

(c) $\displaystyle\sum_{n=2}^{\infty} \ln\left(\frac{n^2+n}{4-9n+5n^2}\right)$

5. (10 points) Find all possible values for r so that $\displaystyle\sum_{n=2}^{\infty} 5r^n = \frac{4}{3}$.

6. (13 points) Indicate whether the following are **Always True** or **Sometimes False** by circling your answer below the statement. If the answer is Sometimes False, provide an example below to show why it's Sometimes False. No further justification is necessary.

   (i) If $\{a_n\}$ diverges so does $\{|a_n|\}$

   (ii) If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ diverges.

   (iii) If $\{a_n\}$ converges, it is bounded.

   (iv) If $\{a_n\}$ converges, it is monotonic.

   (v) If $\displaystyle\sum_{n=1}^{\infty} \left(a_n \left(\frac{2}{3}\right)^n\right)$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 0$.