

1. (8 points) Consider the rational function $r(x) = \frac{4x^3 + 8x^2 + 13}{(2x + 3)^3(x - 9)(x^2 + 2x + 7)^2}$. Write out the form of the partial fraction decomposition, but do not solve for the coefficients.

2. (36 pts) Evaluate the following integrals.

(a) $\int (\cos \theta + 3 \tan \theta)^2 d\theta$

(b) $\int x^2 \ln(x + 4) dx$

(c) $\int_3^6 \frac{\sqrt{x^2 - 9}}{x^3} dx$

3. (20 pts) Consider the region \mathcal{R} bounded by $y = x$ and $y = 2 - x^2$ for $x < 0$ (to the left of the y -axis).

(a) Use the grid below to sketch and shade the region \mathcal{R} .

(b) Set up but do not evaluate integrals to determine each of the following:

I. The area of \mathcal{R} .

II. The volume of the solid when \mathcal{R} is rotated about $y = 2$.

III. The volume of the solid when \mathcal{R} is rotated about the y -axis.

4. (20 pts) Determine whether the following integrals are convergent or divergent. Explain your reasoning fully for each integral. (If the integral converges, find its value. If you use the Comparison Test, state this and fully illustrate any inequality that is part of your argument.)

(a) $\int_2^\infty \frac{x}{\sqrt{x^3 - 1}} dx$

(b) $\int_0^2 \frac{x - 2}{\sqrt{x}} dx$

5. (16 points) Consider the table below which provides values of $f(x)$ and some of its derivatives. You may also assume that $f'''(x) > 0$ for all $x \geq 2$. Use this information to answer the questions below the table.

x	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{2}$
$f'(x)$	3	$\frac{1}{2}$	$-\frac{1}{9}$	$-\frac{3}{7}$	$-\frac{2}{3}$	$-\frac{3}{4}$	$-\frac{4}{5}$
$f''(x)$	$-\frac{5}{2}$	-3	$-\frac{12}{5}$	$-\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{5}$

- (a) Write out an approximation for $\int_2^4 f(x) dx$ using the trapezoidal rule with $n = 4$. (You DO NOT need to simplify your final answer. Your answer should be in a form that *could* be directly input into a calculator.)
- (b) What is the minimum value of n where the trapezoidal rule would give an approximation of $\int_2^4 f(x) dx$ whose error is less than or equal to 10^{-3} ?