1. (32 pts) The following three problems are not related.

   (a) Evaluate $\displaystyle\int \tan^2(3x)\,dx$.

   (b) i. Evaluate $\displaystyle\int_0^1 \frac{x^2}{\sqrt{4-x^2}}\,dx$.

   ii. Does the improper integral $\displaystyle\int_0^2 \frac{x^2}{\sqrt{4-x^2}}\,dx$ converge or diverge? If it converges, to what does it converge? Justify your answer using limits.

   (c) Does the series $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{\sqrt{n}}{2+n}\right)$ converge or diverge? If it converges, to what does it converge?

2. (25 pts) Consider the integral $\displaystyle\int_0^1 \ln(1+x)\,dx$.

   (a) Evaluate the integral.

   (b) Use the Maclaurin series for $\ln(1+x)$ to find a series representation for the integral.

   (c) Apply the Alternating Series Estimation Theorem to the series found in part (b). Find an approximation for the value of the integral with an error less than $\frac{1}{15}$. (You may assume that the conditions of the theorem are satisfied.)

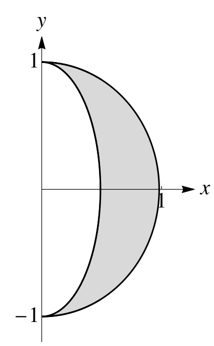
3. (35 pts) The shaded region $\mathcal{R}$ bounded by the curve $x = \frac{1}{2}\sqrt{1-y^2}$ and the unit circle forms a "crescent moon" in quadrants I and IV, as shown below.

   (a) Set up integrals to find the following quantities. Simplify integrands but otherwise do not evaluate the integrals.

   I. Area of the region $\mathcal{R}$ (using the Cartesian area formula)

   II. Volume of the solid generated by rotating the region $\mathcal{R}$ about the y-axis using the Disk-Washer Method.

   III. Area of the surface generated by rotating the curve $x = \frac{1}{2}\sqrt{1-y^2}$ about the line $y = -2$.



   (b) The curve $x = \frac{1}{2}\sqrt{1-y^2}$ forms a semi ellipse (half of an ellipse), as shown in the previous figure.

   I. Find a parametric representation for the semi ellipse in terms of the trigonometric functions $\sin t$ and $\cos t$. Specify the t interval.

   II. Set up but do not evaluate an integral to find the area of the region $\mathcal{R}$ using the polar area formula. (*Hint:* Find polar representations for the two curves.)

4. (18 pts) Suppose $\sum_{n=1}^{\infty} a_n$ is a series such that the corresponding sequence of partial sums is given by

$$s_n = \sum_{i=1}^{n} a_i = \ln\left(\frac{3n}{n+1}\right).$$

Answer the following questions. Be sure to justify your answers.

(a) Find the values of a_1 and a_2, the first two terms of the series. Simplify your answers.

(b) Does $\sum_{n=1}^{\infty} a_n$ converge or diverge? If it converges, to what does it converge?

(c) Does a_n converge or diverge? If it converges, to what does it converge?

5. (16 pts) Consider the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2x+6)^n}{7^n}$.

(a) Find the center a and the radius of convergence R of the series.

(b) Find the sum of the series. Simplify your answer.

6. (16 pts)

(a) Suppose the position of a particle at time t is given by the curve C_1:

$$x_1 = t - \sin t, \quad y_1 = 1 - \cos^2 t, \quad 0 \le t \le 2\pi,$$

and the position of a second particle is given by the curve C_2:

$$x_2 = t - \cos t, \quad y_2 = 1 - \sin^2 t, \quad 0 \le t \le 2\pi.$$

Are the particles ever at the same place at the same time t? If so, find the value(s) of t when these collision points occur. If not, explain why not.

(b) Find the slope of the line tangent to curve C_1 at $t = \frac{\pi}{3}$.

7. (8 pts) Use the r-θ graph shown below to sketch the corresponding polar curve in the xy-plane.