

1. (20 pts) Are the following series absolutely convergent, conditionally convergent, or divergent? Justify your answers and name any tests that you use.

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{5n+2}}$$

(b)
$$\sum_{n=0}^{\infty} \frac{(n!)^3}{(3n)!}$$

2. (20 pts) Consider the integral $\int \arctan(5x) dx$.

(a) Find a Maclaurin series representation for the integral. Write your answer in sigma notation and simplify.

(b) What is the radius of convergence of the series found in part (a)? Explain your answer.

(c) Find a series representation for $\int_0^{\frac{1}{7}} \arctan(5x) dx$. Write your answer in sigma notation and simplify.

3. (24 pts) Let $f(x) = \sqrt{x}$.

(a) Find the Taylor polynomial $T_2(x)$ for $f(x)$, centered at $a = 1$.

(b) Use $T_2(x)$ to approximate the value of $\sqrt{\frac{11}{10}}$. Simplify your answer.

(c) Use Taylor's Formula to find an error bound for the approximation found in part (b). Simplify your answer.

4. (20 pts) Let $g(x) = x^3 e^{2x}$.

(a) Find the Maclaurin series for $g(x)$. Write your answer in sigma notation and simplify.

(b) What is the value of $g^{(13)}(0)$? You do not need to simplify your answer.

(c) Find the sum of $\sum_{n=0}^{\infty} \frac{2^{2n+3}}{n!}$.

5. (16 pts) Consider the parametric curve defined by $x = e^{t/2}$, $y = e^t$.

(a) Sketch the curve. Find and label all intercepts. Indicate with an arrow the direction of motion as t increases.

(b) Eliminate the parameter to find a Cartesian equation $y = f(x)$ of the curve.