

1. (24 pts) Consider the region \mathcal{R} in the first quadrant bounded above by $y = \cosh x$, below by $y = 1$, and on the right by $x = \ln 2$.

(a) Sketch and shade the region \mathcal{R} .

(b) Set up but do not evaluate integrals to determine each of the following:

I. The volume of the solid generated by rotating \mathcal{R} about the y -axis.

II. The volume of the solid generated by rotating \mathcal{R} about the line $y = 3$.

III. The length of the curve $y = \cosh x$ for $0 \leq x \leq \ln 2$. (Simplify the integrand, eliminating all square roots.)

2. (14 pts) Find the surface area when $y = 6\sqrt{x+7}$ from $x = 0$ to 9 is rotated about the x -axis. Evaluate the corresponding integral.

3. (18 pts) The following two problems are not related.

(a) Masses $m_1 = 3$ and $m_2 = k$ are located at $(k, -8)$ and $(1, 3k)$, respectively. The moment M_x of the system equals $6k$. What is the value of k ?

(b) Solve the initial value problem. Simplify your answer and express it in the form $y = f(x)$.

$$\frac{\csc^2 x}{y} \cdot \frac{dy}{dx} = \sec^2 x, \quad y\left(\frac{\pi}{4}\right) = e^{-\pi/4}$$

4. (24 pts) The following two problems are not related.

(a) Does $\sum_{n=1}^{\infty} \frac{5+3^n}{4^n}$ converge or diverge? If it converges, find the sum.

(b) i. Is $\left\{ \frac{\pi n}{1-2n} \right\}$ monotonic? Justify your answer.

ii. Does $\sum_{n=1}^{\infty} \sin\left(\frac{\pi n}{1-2n}\right)$ converge or diverge? If it converges, find the sum.

5. (20 pts) Suppose $\sum_{n=1}^{\infty} a_n$ is a series such that the corresponding sequence of partial sums is given by

$$s_n = \arctan\left(\frac{n^2+2}{7-3n}\right).$$

(a) Find a_1 and a_2 , the first two terms of the series. You may leave your answer unsimplified.

(b) Does $\sum_{n=1}^{\infty} a_n$ converge? If so, to what does it converge? Justify your answer.

(c) Does $\{a_n\}$ converge? If so, to what does it converge? Justify your answer.