1. (26 pts) Evaluate the integral.

   (a) $\displaystyle\int \frac{2x^2 - 3x + 10}{x^3 + 5x}\,dx$

   (b) $\displaystyle\int \frac{1}{(x^2-1)^{3/2}}\,dx$

2. (20 pts) This problem has three parts.

   Let $f(x) = 1 + \ln\left(\dfrac{x}{x+1}\right)$. Consider the integral $\displaystyle\int_1^4 f(x)\,dx$.

   (a) Estimate the value of the integral using T_3, the trapezoidal approximation with $n = 3$ subintervals. Fully simplify your answer by combining logarithms.

   (b) Given that $-\frac{3}{4} \le f''(x) < -\frac{1}{50}$ for $1 \le x \le 4$, how large should n be to ensure that the approximation error for T_n is within 10^{-4}? Simplify your answer.

   (c) Is the T_3 approximation found in part (a) an underestimate or overestimate? Justify your answer. (*Hint:* It is not necessary to find the exact value of the integral.)

3. (30 pts) The following three problems are not related.

   (a) Find the value of $\sin^{-1}\left(\cot\left(\cos^{-1}\left(1/\sqrt{5}\right)\right)\right)$.

   (b) Evaluate $\displaystyle\int_0^\infty 6xe^{-2x}\,dx$. Justify any indeterminate limits.

   (c) Does $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}\,(1+x^5)}$ converge or diverge? Justify your answer.

4. (24 pts) Consider the region $\mathcal{R}$ bounded by $y = 4\sqrt{x}$, $x = 0$, and $y = 1$.

   (a) Sketch and shade the region $\mathcal{R}$.

   (b) Set up but <u>do not evaluate</u> integrals to determine each of the following:

   I. The area of $\mathcal{R}$ using integration with respect to x.

   II. The area of $\mathcal{R}$ using integration with respect to y.

   III. The volume of the solid when $\mathcal{R}$ is rotated about $y = 1$ using the disk method.