APPM 1360 | Final Exam | Spring 2021

This exam is worth 150 points and has 7 questions. Solutions must be written on blank paper. Notes, papers, calculators, cell phones, and other electronic aids are not permitted.

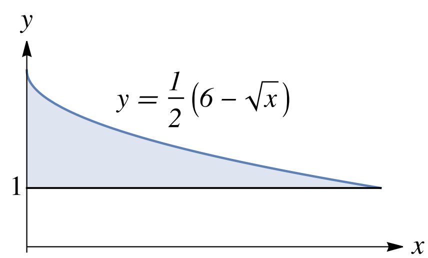
Show all work and simplify your answers. Answers without proper justification will receive no credit unless the problem explicitly states otherwise.

When done with the test, scan and upload your solutions to Gradescope. Be sure to **match each page to the corresponding problem numbers**.

1. (24 pts) Evaluate the following integrals. Be sure to simplify your answers.

   (a) $\displaystyle\int_1^\infty \frac{13}{(3x-1)(x+4)}\,dx$ (b) $\displaystyle\int x\sec^2 x\,dx$

2. (23 pts) Consider the region $\mathcal{R}$, shown below, bounded by $y=\frac{1}{2}\left(6-\sqrt{x}\right)$, $y=1$, and the y-axis.



   Set up but do not evaluate integrals to find the following quantities.

   (a) The volume of the solid obtained by rotating $\mathcal{R}$ about the line $y=4$ using the Disk-Washer Method.

   (b) The volume of the solid obtained by rotating $\mathcal{R}$ about the line $y=4$ using the Cylindrical Shells Method.

   (c) The area of the surface generated by rotating the upper border of the region about the line $x=0$.

3. (14 pts) Does the sequence or series converge? If so, what does it converge to? Justify your answer and name any tests or theorems you use.

   (a) $\left\{\dfrac{\sin\left(\pi+\frac{1}{n}\right)}{\tan\left(\pi+\frac{1}{n}\right)}\right\}$ (b) $\displaystyle\sum_{n=1}^{\infty}\ln\left(\frac{9n}{9+10n}\right)$

MORE PROBLEMS ON THE NEXT PAGE

4. (26 pts) A power series representation for a function $g(x)$ is $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^{2n}}$.

   (a) Find the center and radius of convergence of the series.

   (b) Use the power series for $g(x)$ to find a series representation for $\displaystyle\int_{1/2}^{1} g(x)\,dx$.

   Write your answer in sigma notation.

   (c) Find the sum of the $g(x)$ power series and simplify.

5. (17 pts) The nth derivative of a function $f(x)$ is $f^{(n)}(x) = \dfrac{-2^n(n-1)!}{(3-2x)^n}$ for $n \geq 1$.

   (a) Find the Maclaurin series for $f(x)$ given that $f(0) = 0$. Write your answer in sigma notation.

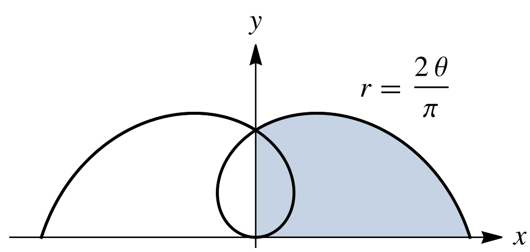
   (b) Suppose $T_3(x)$, the third degree Taylor polynomial of f, centered at 0, is used to estimate the value of $f\left(\frac{1}{2}\right)$. Use Taylor's Remainder Formula to find an error bound for the approximation.

6. (22 pts) Consider the parametric curve given by $x = 4-2t,\ y = e^t + e^{-t},\ 0 \leq t \leq 3$. Fully simplify your answers to the following questions.

   (a) Use the parametric slope formula to find the tangent slope at $x = 4 - 2\ln 2$.

   (b) Use the parametric arc length formula to find the length of the curve.
   $\big($*Hint:* $(dx/dt)^2 + (dy/dt)^2$ is a perfect square.$\big)$

7. (24 pts) Consider the polar curve $r = \dfrac{2\theta}{\pi}$, $-\pi \leq \theta \leq \pi$, shown below.



   (a) Find equations of the lines tangent to the curve at $x = 0$, $y = 1$.

   (b) Set up but do not evaluate integrals to find the following quantities. Simplify the integrands.

      i. The shaded area in the first quadrant bounded by the curve and the positive x and y axes.

      ii. The length of the entire curve

END OF TEST

Trigonometric identities

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin^2(x) = \tfrac{1}{2}\left(1 - \cos(2x)\right)$$
$$\cos^2(x) = \tfrac{1}{2}\left(1 + \cos(2x)\right)$$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(u/a) + C$$

Error Bounds for Trapezoidal and Midpoint Rules

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \qquad |E_M| \le \frac{K(b-a)^3}{24n^2}$$

Center of Mass Integrals

$$M = \int_a^b \rho(f(x) - g(x))\,dx$$
$$M_y = \int_a^b \rho x(f(x) - g(x))\,dx$$
$$M_x = \int_a^b \frac{1}{2}\rho\left[(f(x))^2 - (g(x))^2\right]dx$$
$$\bar{x} = \frac{M_y}{M} \text{ and } \bar{y} = \frac{M_x}{M}$$

Frequently Used Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad R = 1$$
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$
$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$$
$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$$
$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad R = 1$$
$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$$
$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \qquad R = 1$$

Taylor Series and Taylor's Formula

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

Ellipse and Hyperbola

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ or } \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Area

$$A = \int_a^b g(t)f'(t)\,dt \qquad x = f(t),\;\; y = g(t)$$
$$A = \int_\alpha^\beta \frac{1}{2}r^2\,d\theta$$

Volume

$$V = \int_a^b A(x)\,dx \qquad V = \int_a^b \pi\left(R^2 - r^2\right)dx$$
$$V = \int_a^b 2\pi r h\,dx$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$
$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$
$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

Surface Area

$$S = \int_a^b 2\pi r\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$