APPM 1360 **Exam 3** **Spring 2021**

This exam is worth 100 points and has 6 questions. Solutions must be written on blank paper. Notes, papers, calculators, cell phones, and other electronic aids are not permitted.

Show all work and simplify your answers. Answers without proper justification will receive no credit unless the problem explicitly states otherwise.

When done with the test, scan and upload your solutions to Gradescope. Be sure to **match each page to the corresponding problem numbers**.

---

1. (24 pts) Are the following series absolutely convergent, conditionally convergent, or divergent? Justify your answer and name any tests or theorems you use.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{5^{2n}}{n^2\, 9^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{n} - e^{-n}\right)$ (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\arctan(n)}{n^2+1}$

2. (11 pts) Consider the series $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n} + \frac{1}{n+1}\right)$.

   (a) Find an expression for s_n, the nth partial sum of the series.

   (b) Use the partial sum, s_n, to determine if the series converges or diverges. If it converges, find the sum.

3. (18 pts) The function $f(x)$ has the power series representation $\displaystyle\sum_{n=0}^{\infty} \frac{2^n\, n!}{(2n)!}(x+7)^n$.

   (a) Find the center, radius of convergence, and interval of convergence of the series.

   (b) Find the value of $f^{(17)}(-7)$. You may leave your answer unsimplified.

4. (14 pts) Consider the following questions about the function $g(x) = \dfrac{2}{3x+1}$.

   Write your answers using sigma notation. Do not use the binomial series formula.

   (a) Find a power series representation for $g(x)$, centered at 0.

   (b) Use your answer from part (a) to find a power series representation for $\dfrac{-6x^3}{(3x+1)^2}$.

MORE PROBLEMS ON THE NEXT PAGE

5. (16 pts)

   (a) Use a MacLaurin series to find a series representation for $\sin\left(\frac{1}{10}\right)$. Write this series using sigma notation.

   (b) Determine the approximate value of $\sin\left(\frac{1}{10}\right)$ with an error less than 10^{-6} by applying the Alternating Series Estimation Theorem to the series from part (a). Assume that the conditions of the theorem are met. Use the minimum number of terms needed and simplify your answer.

   (c) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{10\pi}{(2n+1)!}$.

6. (17 pts) A ball is dropped from a height of h_0 meters. The ball bounces indefinitely, with each bounce losing 20% of its previous height.

   (a) Write a recursive expression for h_n, the height of the ball produced by the nth bounce, $n \geq 1$.

   (b) Use sigma notation to find a non-recursive representation for the total distance traveled by the ball.

   (c) Calculate the total distance.

END OF TEST

Frequently Used Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad R = 1$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$$

Taylor Series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Trigonometric identities

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin^2(x) = \tfrac{1}{2}\left(1 - \cos(2x)\right)$$
$$\cos^2(x) = \tfrac{1}{2}\left(1 + \cos(2x)\right)$$