APPM 1360 **Exam 2** **Spring 2021**

This exam is worth 100 points and has 4 questions. Solutions must be written on blank paper. Notes, papers, calculators, cell phones, and other electronic aids are not permitted.

Show all work and simplify your answers. Answers without proper justification will receive no credit unless the problem explicitly states otherwise.

When done with the test, scan and upload your solutions to Gradescope. Be sure to **match each page to the corresponding problem numbers**.

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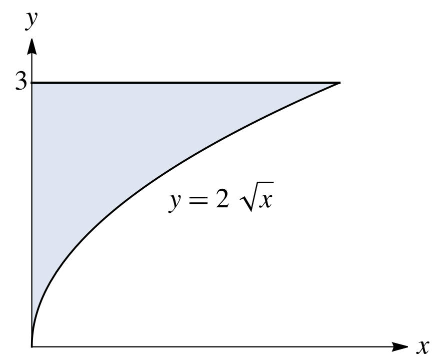
1. (32 pts) Consider the region bounded by $y = 2\sqrt{x}$, $x = 0$, and $y = 3$, shown below. Set up but do not evaluate integrals to determine each of the following quantities.



   (a) The area of the region using integration with respect to y.

   (b) The volume of the solid generated when the region is rotated about $x = 5$ using the disk/washer method.

   (c) The volume of the solid generated when the region is rotated about $x = 5$ using the shell method.

   (d) The surface area generated by rotating the lower curve $y = 2\sqrt{x}$ about the line $x = -1$.

2. (20 pts) The arc length integral for a function $y = f(x)$ on $[0, a]$ is $\displaystyle\int_0^a \sqrt{\frac{4+x}{4}}\, dx$.

   (a) Evaluate the integral to find the length of the curve on $[0, a]$ for $a \geq 0$.

   (b) Find the function $f(x)$ given that $f(9) = 1$ and $f'(x) \geq 0$ for $x \geq 0$.

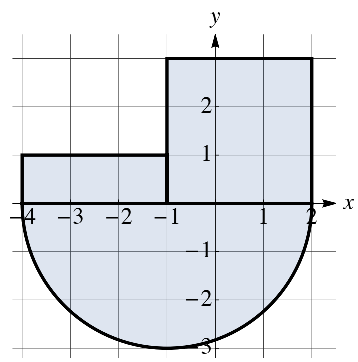
MORE PROBLEMS ON THE NEXT PAGE

3. The following two problems are not related.

   (a) (14 pts) Solve for y in the differential equation $\left(x^2y+y\right)\dfrac{dy}{dx}=x^2$.

   (b) (10 pts) Consider the region $\mathcal{R}$, shown below, composed of a semicircular region and two rectangles. The centroid of the semicircular region is located at $\left(-1,-\frac{4}{\pi}\right)$. Assume that R has uniform density ρ. Find the following quantities <u>without</u> using integration.

      i. M, the mass of $\mathcal{R}$
      ii. M_x, the moment of $\mathcal{R}$ about the x-axis
      iii. $\overline{y}$, the y-coordinate of the centroid of $\mathcal{R}$



4. The following problems are not related.

   (a) (14 pts) Determine whether the sequence converges or diverges.

      i. $a_n=\dfrac{\sqrt[3]{n}}{\ln(300n)}$

      ii. $\left\{\dfrac{\cos(n!)}{\sqrt{n}}\right\}$

   (b) (10 pts) The White Rabbit has swallowed a pill that immediately causes him to grow in height by 10% in 30 seconds, then shrink in height by 10% the next 30 seconds, then grow in height by 10% the next 30 seconds, etc., continuing to alternately increase and decrease his height by 10% every 30 seconds.

   Let h_0 represent the White Rabbit's normal height, and let h_n represent his height <u>n minutes</u> after he swallows the pill, for $n\geq 1$. Simplify your answers to the following problems.

      i. Find a recursive expression for h_n, for $n\geq 1$.
      ii. Find a non-recursive expression for h_n, for $n\geq 1$.
      iii. If the White Rabbit continues to grow and shrink indefinitely, what will his eventual height be? Justify your answer.

END OF TEST

Trigonometric identities

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin^2(x) = \tfrac{1}{2}\left(1 - \cos(2x)\right)$$
$$\cos^2(x) = \tfrac{1}{2}\left(1 + \cos(2x)\right)$$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(u/a) + C$$

Center of Mass Integrals

$$M = \int_a^b \rho\left(f(x) - g(x)\right)dx$$
$$M_y = \int_a^b \rho x\left(f(x) - g(x)\right)dx$$
$$M_x = \int_a^b \frac{1}{2}\rho\left[(f(x))^2 - (g(x))^2\right]dx$$
$$\bar{x} = \frac{M_y}{M} \text{ and } \bar{y} = \frac{M_x}{M}$$