APPM 1360 **Exam 1** **Spring 2021**

This exam is worth 100 points and has 3 questions. Solutions must be written on blank paper. Notes, papers, calculators, cell phones, and other electronic aids are not permitted.

Show all work and simplify your answers. Answers without proper justification will receive no credit unless the problem explicitly states otherwise.

When done with the test, scan and upload your solutions to Gradescope. Be sure to **match each page to the corresponding problem numbers**.

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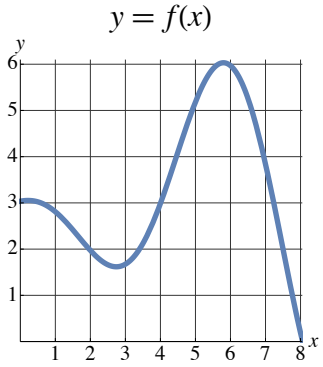
1. (36 pts) Evaluate the integral.

   (a) $\displaystyle\int \left(\tan^2\theta + \tan^4\theta\right) d\theta$ (b) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$

   (c) $\displaystyle\int_0^{\pi} 4x\cos^2 x\, dx$ (*Hint:* First find an antiderivative for $4\cos^2 x$.)

2. The following two problems are not related.

   (a) (16 pts) Consider a function $f(x)$ whose graph is given below.

   Suppose we wish to approximate the value of $\displaystyle\int_0^8 f(x)\, dx$.

   i. Find the T_4 trapezoidal approximation.
   ii. Find the M_2 midpoint approximation.
   iii. Is M_2 an underestimate or overestimate? Explain your answer.

   (b) (10 pts) Consider $\displaystyle\int_0^1 (x+3)e^{-2x}\, dx$. How large should n be to ensure that an M_n approximation of this integral using the midpoint rule will have an error less than 10^{-4}?

   Hint: The first derivative of $(x+3)e^{-2x}$ is $-(2x+5)e^{-2x}$.

MORE PROBLEMS ON THE NEXT PAGE

3. The following three problems are not related.

   (a) (10 pts) Find the partial fraction decomposition of $\dfrac{x^2}{x^2-5x+6}$ including the values of the coefficients.

   (b) (12 pts) Determine whether $\displaystyle\int_0^\infty \frac{\sin^2 x}{4x+e^x}\,dx$ is convergent or divergent. Fully justify your answer.

   (c) (16 pts) Consider the integral $\displaystyle\int_k^\infty \frac{7}{x^2-x-12}\,dx$ where k is a constant.

      i. Evaluate the integral. Fully justify your answer.

      Hint: The decomposition of $\dfrac{7}{x^2-x-12}$ is $\dfrac{1}{x-4}-\dfrac{1}{x+3}$.

      ii. Are there any values of k for which the integral is convergent? If so, find all such values of k. If not, explain why there are none.

END OF TEST

Trigonometric identities

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin^2(x) = \tfrac{1}{2}\left(1-\cos(2x)\right)$$
$$\cos^2(x) = \tfrac{1}{2}\left(1+\cos(2x)\right)$$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}(u/a) + C$$

Error Bounds for Trapezoidal and Midpoint Rules

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \qquad |E_M| \le \frac{K(b-a)^3}{24n^2}$$