On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- Show all work and simplify your answers! Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

1. (33 pts)
   (a) Find the partial fraction decomposition of \( \frac{9}{x^3 + 6x^2 + 9x} \) including the values of the coefficients.
   (b) Evaluate the following integrals.
      i. \( \int \frac{1}{\sqrt{25 + x^2}} \, dx \)
      ii. \( \int_0^{1/3} \arctan(3x) \, dx \)

2. (17 pts)
   (a) Approximate \( \int_0^1 \sin(\pi x) \, dx \) using the Trapezoidal Rule with three subintervals. Simplify your answer.
   (b) Use the formula for \( |E_T| \) to find an error bound for the approximation.

3. (22 pts)
   (a) Determine whether the integral \( \int_0^1 \frac{2 \cos^2 x}{x^{2/3}} \, dx \) converges or diverges.
   (b) Suppose the function \( f \) is continuous for all real numbers. Given that \( \int_{-\infty}^8 f(x) \, dx \) converges and \( \int_8^\infty f(x) \, dx \) diverges, state whether each of the following integrals is Convergent or Divergent. If the convergence cannot be determined, write Indeterminate. No justification is necessary.
      A. \( \int_{-\infty}^0 f(x) \, dx \)
      B. \( \int_{-\infty}^{100} f(x) \, dx \)
      C. \( \int_8^\infty [f(x)]^2 \, dx \)
      D. \( \int_{-\infty}^\infty f(x) \, dx \)
      E. \( \int_{-10}^{100} \left( 1000 + f(x) \right) \, dx \)
      F. \( \int_{-\infty}^8 \left( f(x) - \frac{1}{2} \right) \, dx \)

TURN OVER—More problems on the back!
4. (28 pts)

(a) Set up (but do not evaluate) an integral to compute the area of the region in the first quadrant bounded by \( x = y^3 + 3y \) and \( y = x/7 \).

(b) Consider the region in the first quadrant bounded by \( y = \ln(x + 1), y = 2 \), and the \( y \)-axis.

i. Sketch the region.

ii. Suppose the region is rotated about the specified line. Use the disk/washer method to set up (but do not evaluate) an integral to find the volume of the generated solid.

   A. about \( y = 5 \)  
   B. about the \( y \)-axis

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**Trigonometric identities**

\[
\begin{align*}
\sin(2x) &= 2 \sin(x) \cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin^2(x) &= \frac{1}{2} (1 - \cos(2x)) \\
\cos^2(x) &= \frac{1}{2} (1 + \cos(2x))
\end{align*}
\]

**Inverse Trigonometric Integral Identities**

\[
\begin{align*}
\int \frac{du}{\sqrt{a^2 - u^2}} &= \sin^{-1} \left( \frac{u}{a} \right) + C \\
\int \frac{du}{a^2 + u^2} &= \frac{1}{a} \tan^{-1} \left( \frac{u}{a} \right) + C \\
\int \frac{du}{u \sqrt{u^2 - a^2}} &= \frac{1}{a} \sec^{-1} \left( \frac{u}{a} \right) + C
\end{align*}
\]

**Error Bounds for Trapezoidal and Midpoint Rules**

\[
\begin{align*}
|E_T| &\leq \frac{K(b - a)^3}{12n^2} \\
|E_M| &\leq \frac{K(b - a)^3}{24n^2}
\end{align*}
\]