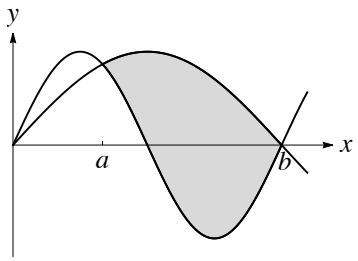
1. (28 pts) Consider the shaded region $\mathcal{R}$ shown below, bounded by $y = \sin(x)$ and $y = \sin(2x)$.



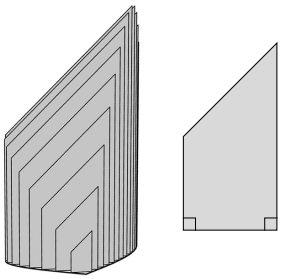
   (a) The shaded region extends from $x = a$ to $x = b$. Determine the values of a and b.

   (b) Set up (but do not evaluate) integrals to find the following quantities.

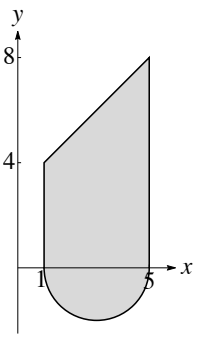
      i. The volume of the solid obtained by rotating $\mathcal{R}$ about the line $x = b$.

      ii. The area of the surface generated by rotating $y = \sin(2x)$, $a \le x \le b$, about the horizontal line tangent to the upper boundary of region $\mathcal{R}$.

      iii. The volume of the solid with $\mathcal{R}$ as the base and cross-sections perpendicular to the x-axis in the shape of right trapezoids. (Side view of sample cross-sections shown.) The parallel sides of each trapezoid extend up out of region $\mathcal{R}$, with one side twice as long as the other. A third side is in region $\mathcal{R}$ and has length equal to the shorter parallel side.



2. (8 pts) Region $\mathcal{S}$, shown at right, is formed by joining a right trapezoid to a semi-circular region. The vertices of the trapezoid are $(1, 0)$, $(5, 0)$, $(5, 8)$, and $(1, 4)$. Set up (but do not evaluate) an integral to find M_y, the moment of region $\mathcal{S}$ about the y-axis. Assume $\mathcal{S}$ has uniform density ρ.



3. (14 pts) Solve for y in the differential equation given the initial condition $y(0) = e$.

$$\frac{dy}{dx} = y\left(1 + (\ln y)^2\right)$$

4. (32 pts) The following four problems are not related. Justify your answers to the following questions.

   (a) Does $a_m = m^2 \sin\left(3/m^2\right)$ converge? If so, what does it converge to?

   (b) Is the sequence $b_n = \dfrac{\sqrt{n} + 3}{\sqrt{n} + 4}$ monotonic?

   (c) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\left(1 + (\ln n)^2\right)}$ converge? (*Hint:* You may refer to other solutions in this exam.)

(d) Are there values of k for which $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3+n}}{k^{-n}}$ converges? If so, determine all such values of k and find the sum of the series. If not, explain why.

5. (18 pts) Justify your answers to the following questions.

(a) Consider the series $\displaystyle\sum_{n=1}^{\infty} \left(\ln\left(\tfrac{1}{n^2}\right) - \ln\left(\tfrac{1}{n}\right)\right)$.

i. Find s_3, the third partial sum of the series. Write your answer as a single logarithm.

ii. Does the series converge? If so, what does it converge to?

(b) Consider the series $\displaystyle\sum_{n=1}^{\infty} \left(\cos\left(\tfrac{\pi}{n}\right) - \cos\left(\tfrac{\pi}{n+1}\right)\right)$.

i. Find s_3, the third partial sum of the series. Simplify your answer.

ii. Does the series converge? If so, what does it converge to?