

1. (36 pts) Evaluate the integral.

(a)  $\int \frac{3x^2 - 2x + 12}{x^3 + 4x} dx$

(b)  $\int 2x \arctan(x) dx$

(c)  $\int \frac{dx}{x^2 \sqrt{x^2 - 9}}$

2. (16 pts) Consider the integral  $\int_0^{3\pi/4} x \sin^2(x) dx$ .

(a) Estimate the integral using the trapezoidal approximation  $T_3$ . Fully simplify your answer.

(b) Estimate the error  $|E_T|$  in the approximation  $T_3$ . Leave your answer unsimplified.

*Hint:* Let  $f(x) = x \sin^2(x)$ . Then  $f'(x) = x \sin(2x) - \frac{1}{2} \cos(2x) + \frac{1}{2}$ .

3. (24 pts) Consider the region bounded above by  $y = 2 + \ln x$ , below by the line  $y = 2$ , and on the right by the line  $x = e$ .

(a) Sketch and shade the region.

(b) Set up (but do not evaluate) integrals to find the following quantities:

i. The area of the region, integrating with respect to  $x$ .

ii. The area of the region, integrating with respect to  $y$ .

iii. The volume of the solid generated by rotating the region about the line  $y = -1$ .

4. (24 pts) The following problems are not related.

(a) Determine whether  $\int_1^\infty \frac{dx}{x \arctan(x)}$  is convergent or divergent. Justify your answer.

(b) Evaluate  $\int_0^{100} \frac{dx}{(x-a)^2}$  for  $0 < a < 100$ . Is the integral convergent or divergent?