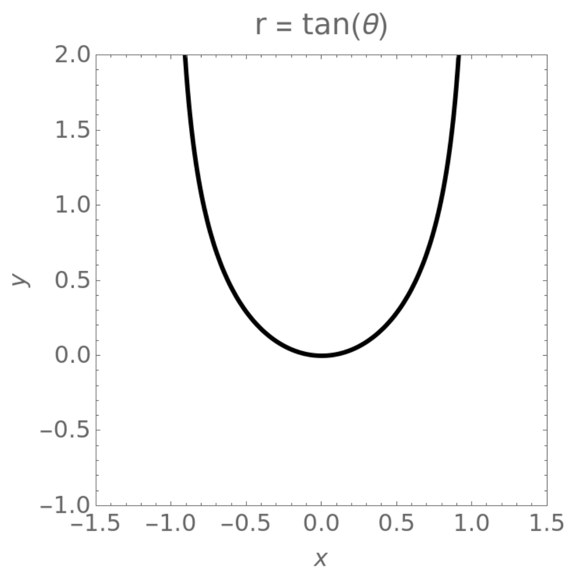
1. (34 points) Find the requested information. The problems are unrelated.
   (a) Evaluate $\displaystyle\int \frac{\tan^{-1}(x)}{x^2}\,dx$ (Hint: Start with IBP)
   (b) Find y as a function of x given that $\dfrac{dy}{dx} = 2x\sqrt{1-y^2}$ and $y(0) = 1$
   (c) Find the sum of the series $-\dfrac{x^6}{3!} + \dfrac{x^{10}}{5!} - \dfrac{x^{14}}{7!} + \cdots$
   (d) For what values of θ does the series $\displaystyle\sum_{n=0}^{\infty} \sin^2(\theta)\cos^{2n}(\theta)$ converge? Find the sum for those values of θ.
2. (16 points) Decide whether the following quantities are convergent or divergent. Explain your reasoning and name any test you use.
   (a) The sequence given by $a_n = \left(1 - \dfrac{\ln(3)}{n}\right)^n$, for $n = 1, 2, \ldots$
   (b) $\displaystyle\int_1^{\infty} \frac{1}{x^2}\sqrt{1+\frac{3}{x^3}}\,dx$
3. (12 points) Consider the series $\displaystyle\sum_{k=1}^{\infty} b_k$. Suppose the n^{th} partial sum of the series is $s_n = 2 - \dfrac{2}{n+1}$.
   (a) What is s_3?
   (b) Find a simple formula for b_n.
   (c) What does $\{b_n\}$ converge to?
   (d) What is the sum of the series $\displaystyle\sum_{k=1}^{\infty} b_k$?
4. (25 points) Recall $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.
   (a) Find the MacLaurin series of the hyperbolic cosine function.
   (b) Find the interval of convergence for the power series from part (a).
   (c) Find $T_3(x)$, the Taylor polynomial of order 3, of the hyperbolic cosine centered at $a = 0$. Use the Taylor Remainder formula to find an upper bound for the absolute error if $T_3(x)$ is used to approximate $\cosh(1)$.
   (d) Use the MacLaurin series (not L'Hôpital!) to evaluate the following limit:
   $$\lim_{x\to 0} \frac{\cosh(x) - 1 - \frac{x^2}{2}}{x^4}.$$
5. (18 points) Suppose $g(x)$ equals the power series $\displaystyle\sum_{n=2}^{\infty} \frac{(n+1)(x+b)^n}{c^{2n}}$, where b and c are constants, and the series has an interval of convergence of $-6 < x < 2$.
   (a) Find the center and radius of convergence of the series.

(b) Evaluate $\int g(x)\,dx$ as a power series.

(c) Given the interval of convergence, find possible values for b and c. Justify your answer using appropriate test(s).

6. (25 points) For this problem, let $r = \tan(\theta)$ for $-\pi/2 < \theta < \pi/2$. The polar graph (in the xy plane) is given below. Answer the following questions.



(a) Find an equation for the tangent line at $\theta = \pi/4$.

(b) Set up an integral to find the length of the curve $r = \tan(\theta)$ for $0 \le \theta \le \pi/4$.

(c) Add a graph of the polar curve $\theta = \pi/4$ to the given graph of $r = \tan(\theta)$.

(d) Find the area of the region bounded by the two curves $\theta = \pi/4$ and $r = \tan(\theta)$

7. (20 points) An equation for an ellipse in parametric form is given by

$$\begin{cases} x = 2\cos t \\ y = 4\sin t \end{cases}, \qquad 0 \le t \le 2\pi.$$

(a) Graph the ellipse labeling the axes and vertices.

(b) Eliminate t to find a Cartesian equation **in standard form** for the ellipse above.

(c) Rotate the area in the upper half-plane between the ellipse and the x-axis around the line $x = 3$ and find the volume. Set up, **but do not evaluate**, an integral to find the volume of the solid.

(d) Rotate the area bounded by the entire ellipse around the line $x = 3$. Set up, **but do not evaluate**, an integral to find the surface area of the solid.