- 1. (24 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. For this problem, and all subsequent problems, explain your work and name any test or theorem that you use.
  - (a)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)}$ (b)  $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$ (c)  $\sum_{j=1}^{\infty} \frac{(3j)^j}{e^j(j+1)^j}$
- 2. (24 points) Consider the power series given by:  $\sum_{k=1}^{\infty} (-1)^k k(6x+3)^k$ 
  - (a) Find the center of the power series.
  - (b) Find the radius of convergence.
  - (c) Find the interval of convergence.
  - (d) Find the sum of the series. Hint: Start with the sum of  $\sum_{k=0}^{\infty} (-1)^k (6x+3)^k$ .
- 3. (20 points) Suppose f(x) has a Taylor series representation, convergent for all x, centered at a = 2,  $f(2) = e^{-6}$ , and  $f^{(n)}(x) = (-1)^n 3^n e^{-3x}$  for n = 1, 2, 3, ...
  - (a) Write down the Taylor series (use sigma notation) and find the sum of the series.
  - (b) Find  $T_2(x)$ , the 2nd order Taylor polynomial.
  - (c) Use Taylor's formula to find an error bound if  $T_2(x)$  is used to approximate f(x) for 2.5 < x < 3.
- 4. (32 points) The following questions are unrelated.
  - (a) Find the sum of the series

$$3 + 3\ln(3) + \frac{3(\ln 3)^2}{2!} + \frac{3(\ln 3)^3}{3!} + \cdots$$

- (b) Let  $f(x) = \left(1 + \frac{x^2}{2}\right)^{1/3}$ . Find  $f^{(20)}(0)$ . Do not simplify your answer.
- (c) Use series multiplication to find the first three nonzero terms of the Maclaurin series for  $e^{x^2} \cos x$ . Clearly box in your answer.
- (d) Graph the following parametric equation drawing an arrow to indicate the direction of travel.

$$x = \sin^2 t, \quad y = \cos^2 t, \quad 0 \le t \le \pi/2.$$

Eliminate the parameter t to find a Cartesian equation for the curve.