1. (22 points) Consider the region $\mathcal{R}$ in the first quadrant bounded by $y=\sin x$ and $y=\frac{2 x}{\pi}$. Set up but do not evaluate the integrals to find the following quantities:
(a) Graph the given equations and shade the region $\mathcal{R}$ labeling the equations and intersection points.
(b) The volume of a solid with a base given by $\mathcal{R}$ and cross-sections perpendicular to the $x$-axis that are isosceles triangles with a height equal to the length of the base. (At each $x$, the base of the isosceles triangle is in $\mathcal{R}$ )
(c) The volume generated by rotating $\mathcal{R}$ about the line $x=3$.
(d) The perimeter of $\mathcal{R}$.
2. (30 points) Three unrelated questions.
(a) (10 points) Find the center of mass of the region bounded between $y=1-x^{2}$ and $y=2\left(1-x^{2}\right)$ for $y>0$. Assume a constant density $\rho_{0}$. The region is shown below.

(b) (8 points) Let $R$ be the radius of the Earth. The gravitational force on a mass $m$ at height $x$ above the Earth's surface has magnitude $F(x)=m g R^{2} /(R+x)^{2}$. How much work is required to move the mass from $x=0$ to a height $x=H$ ? (Assume $H>0 . R, m$, and $g$ are fixed constants.)
(c) (12 points) Find the solution of the differential equation $\frac{d y}{d x}=x y \ln (x)$ with initial condition $y(1)=e$. Express your answer in the form $y=f(x)$.
3. (32 points, 8 points each) Determine whether each of the following converge or diverge. If the quantity converges, find the limit. Explain your work and name any test or theorem that you use.
(a) The sequence given by $a_{n}=\left(1-\frac{\ln 2}{n}\right)^{n}$, for $n=1,2,3, \ldots$
(b) The sequence given by $b_{n}=\sqrt[n]{4^{2+3 n}}$ for $n=1,2,3, \ldots$
(c) $\sum_{n=0}^{\infty} \frac{2^{n-1}+(-1)^{n}}{3^{n}}$
(d) $\sum_{n=1}^{\infty} \frac{n}{n+1} \tan ^{-1} n$
4. (16 points) Consider the sequence given by $a_{n}=\frac{2}{n(n+2)}$ for $n=1,2,3, \ldots$ and the series $\sum_{n=1}^{\infty} a_{n}$.
(a) What is $s_{2}$, the second partial sum of the series $\sum_{n=1}^{\infty} a_{n}$ ? (You need not simplify your answer.)
(b) Find a simple expression for $s_{n}$, the $n^{\text {th }}$ partial sum of the series. Does the sequence $\left\{s_{n}\right\}$ converge? If so, what is its limit?
(c) Does the series $\sum_{n=1}^{\infty} a_{n}$ converge? If so, what is its limit?
