1. ( 36 pts )

Evaluate the following integrals and simplify your answers.
(a) $\int_{0}^{\pi / 2} \sin ^{3}(\theta) \cos ^{4}(\theta) d \theta$
(b) $\int \frac{x^{3}}{\sqrt{x^{2}+9}} d x$
(c) $\int \frac{1}{x(a x+b)} d x$. Assume $a$ and $b$ are positive constants. Express your answer as a single logarithm.
2. ( 30 pts ) Determine whether the following integrals are convergent or divergent. Explain your reasoning fully for each integral. (If the integral converges, find its value, if you can. If you use the Comparison Test, state this and evaluate the integral that you are using for comparison.)
(a) $\int_{0}^{\infty} \frac{1}{x\left((\ln x)^{2}+1\right)} d x$. Note that $\lim _{x \rightarrow 0^{+}} \frac{1}{x\left((\ln x)^{2}+1\right)}=\infty$.
(b) $\int_{1}^{\infty} \frac{1}{x}-\frac{1}{x+1} d x$
(c) $\int_{0}^{1} \frac{1}{(x \sin (x))^{2}} d x$
3. $(18 \mathrm{pts})$ For this problem, let $I=\int_{1}^{2} t \ln t d t$
(a) Calculate the value of $I$.
(b) Estimate $I$ using $T_{2}$, the trapezoidal rule with $n=2$. Express your answer as a single logarithm.
(c) What's the smallest $n$ that will guarantee an error in the Trapezoidal rule of less than $1 / 1200$.
4. (16 pts) For this problem, let $f(x)=\sinh x$ and $g(x)=e^{-x}$. These functions are graphed in the figure below. (Note: $\sinh x=\frac{e^{x}-e^{-x}}{2}$ )
(a) Set up, but don't evaluate, an integral to find the area in the first quadrant bounded between $f$ and $g$. This is the shaded area in the figure below.
(b) Set up, but don't evaluate, an integral to find the volume of the solid generated by rotating the shaded area around the line $y=-1$.


