1. (28 points, 7 points each) Decide whether the following quantities are convergent or divergent. Explain your reasoning and name any test you use.

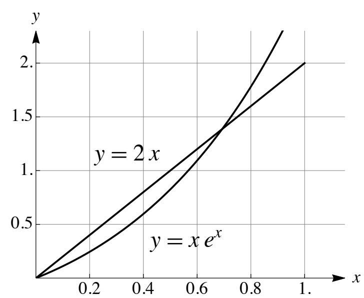
   (a) $\displaystyle\int_0^\infty \frac{e^x}{e^{2x}+1}\,dx$

   (b) The sequence given by $a_n = \dfrac{(\ln n)^{200}}{n}$, for $n = 1, 2, \ldots$

   (c) The sequence given by $a_1 = 2$ and $a_n = -\dfrac{a_{n-1}}{3}$ for $n = 2, 3, \ldots$

   (d) $\dfrac{\pi}{5} + \dfrac{\pi}{10} + \dfrac{\pi}{15} + \cdots$

2. (21 points) Consider the curves $y = 2x$ and $y = xe^x$ shown below. Let $\mathcal{R}$ be the region in the first quadrant bounded above by $y = 2x$ and below by $y = xe^x$.



   (a) Find the (x, y) coordinates of all points of intersection of $y = 2x$ and $y = xe^x$.

   (b) Calculate the area of $\mathcal{R}$.

   (c) Set up, **but do not evaluate**, an integral that gives the volume obtained by rotating $\mathcal{R}$ about the y-axis.

   (d) Set up, **but do not evaluate**, an integral that gives the volume obtained by rotating $\mathcal{R}$ about $y = 2$.

3. (21 points) Consider the series $\displaystyle\sum_{n=1}^{\infty} a_n$ where $a_n = \dfrac{2}{(n+1)(n+2)}$. Let the partial sum $s_n = \displaystyle\sum_{i=1}^{n} a_i$.

   (a) Write the partial fraction decomposition of a_n.

   (b) Find a simple expression for s_n.

   (c) Is $\{s_n\}$ monotonic? Justify your answer.

   (d) Is $\{s_n\}$ bounded? If so, find upper and lower bounds for s_n.

   (e) Does the given series converge? If so, what does it converge to?

4. (16 Points) Suppose a power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ has an interval of convergence of $(2, 8)$. Use this information to answer the following questions. (No justification is necessary for your answers on this problem.)

   (a) Find the center and radius of convergence.

   (b) Is $\displaystyle\sum_{n=0}^{\infty} c_n 4^n$ absolutely convergent, conditionally convergent, divergent, or do you need more information?

(c) For what values of b does $\sum_{n=0}^{\infty} c_n b^n$ converge?

(d) If the Ratio Test is used to find that the interval of convergence is $(2, 8)$, what would $\lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right|$ equal?

5. (20 points) Suppose a function f has $f(1) = \frac{1}{4}$ and the n^{th} derivative of f is $f^{(n)}(x) = \frac{(-1)^n n!}{(x+3)^{n+1}}$ for $n = 1, 2, \ldots, x \neq -3$.

   (a) Find the Taylor series of f centered at $a = 1$.

   (b) Find $T_2(x)$, the Taylor polynomial of order 2, of function $f(x)$ centered at $a = 1$.

   (c) Use the Taylor Remainder formula to find an estimate for the absolute error if $T_2(x)$ is used to approximate $f(2)$.

6. (22 points) The ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ can be parametrized by

$$x(t) = 3\cos(t), \quad y(t) = 5\sin(t) \quad 0 \leq t \leq 2\pi$$

   Using the given parametric equations for the ellipse,

   (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   (b) Find an equation of the tangent line to the ellipse at the point $\left(\frac{3}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$.

   (c) Find the area enclosed by the ellipse.

7. (22 points) Consider the curve $r = \sin(3\theta)$

   (a) Plot the curve on the $r\theta$-plane.

   (b) Plot the curve on the xy-plane.

   (c) Set up, **but do not evaluate**, an integral to find the area outside the circle $r = 1/2$ and inside the curve $r = \sin 3\theta$ in the first quadrant of the xy-plane.

   (d) Set up, **but do not evaluate**, an integral to find the length of the curve $r = \sin(3\theta)$ in the first quadrant of the xy-plane.