1. (28 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. For this problem, and all subsequent problems (except for #5), explain your work and name any test or theorem that you use.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n^2+1}{2n^2+1}\right)^n$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n+1)!}$

   (c) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k-1}(k+3)}{k(k+1)}$

2. (15 points) Consider the power series given by: $\displaystyle\sum_{k=1}^{\infty}(-1)^k(2x-3)^k$

   (a) Find the center of the power series.

   (b) Find the radius of convergence.

   (c) Find the interval of convergence.

   (d) Find the sum of the series.

3. (23 points) Three unrelated questions:

   (a) Suppose the ratio test is applied to a power series $\displaystyle\sum_{n=0}^{\infty}c_n(x-a)^n$. If $\displaystyle\lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right|$ equals a constant $k>0$, what can you conclude about the radius and interval of convergence of the power series?

   (b) Let $f(x)=\left(1+\dfrac{x}{2}\right)^{1/3}$. Find T_2, the Taylor polynomial of order 2, centered at 0. Simplify your answer.

   (c) Write the series in summation notation and then find its sum: $\dfrac{2^2}{3}-\dfrac{2^3}{2\cdot 3^2}+\dfrac{2^4}{3\cdot 3^3}-\dfrac{2^5}{4\cdot 3^4}+\cdots$

4. (22 points) The following questions are related.

   (a) Find the Maclaurin series for $f(x)=\dfrac{\tan^{-1}(2x)}{x}$.

   (b) Use the result from part (a) to find a series solution for $\displaystyle\int f(x)\,dx$.

   (c) Use the first two nonzero terms of the series from part (b) to estimate $\displaystyle\int_0^{0.1} f(x)\ dx$. Find a reasonable estimate for the error. (You need not simplify your answers for this part.)

5. (3 points each) Match the graphs shown below to the following parametric equations. Clearly label each graph with the matching letter (a, b, c, or d). Then draw a single arrow on each graph to indicate the direction in which the curve is traversed. No explanation is required.

$$\begin{array}{llll}
(a) & x = t+1 & y = 2\sqrt{t}+1 & 0 \le t \le 4 \\
(b) & x = e^{-t}+2t & y = e^{t}-2t & -2.5 \le t \le 2.5 \\
(c) & x = 2\sin t & y = t+1 & -6 \le t \le 4 \\
(d) & x = 2\cos t & y = t & -5 \le t \le 5
\end{array}$$