1. Consider the region $\mathcal{R}$ in the first quadrant bounded above by $y = e^{-2x}$, below by $y = e^{-2}$, and the y-axis. For this problem, set up but do not evaluate the integrals to find the requested quantities.

   (a) (5 pts) Graph the given equations and shade the region $\mathcal{R}$. Label the equations and any intersection points.

   (b) (7 pts) The volume of a solid with $\mathcal{R}$ as the base and cross-sections perpendicular to the x-axis that are squares.

   (c) (7 pts) The volume generated by rotating $\mathcal{R}$ about the line $y = -1$ using the shell method.

   (d) (7 pts) The area of the surface generated by rotating the upper curve about the x-axis.

2. Three unrelated questions.

   (a) (8 points) A mass of 1 kg is located at $(0, 0)$, a mass of 2 kg is located at $(a, 0)$, and a mass of 3 kg is at $(0, 5)$. If the x-coordinate of the centroid of this system of masses is $\bar{x} = 1$, find the value of a.

   (b) (8 points) Solve the differential equation $\frac{dy}{dt} = t + y^2 t$ with $y(0) = -1$. Write your answer in the form $y = f(t)$.

   (c) (10 points) A 1600 pound elevator is suspended by a 200 foot cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft? You don't have to fully simplify your answer.

3. (21 points) Determine whether each of the following converge or diverge. If the quantity converges, find the limit. Explain your work and name any test or theorem that you use.

   (a) The sequence given by $a_n = \dfrac{n^2}{e^{3n}}$

   (b) The sequence given by $b_n = n \ln(1 + \pi/n)$

   (c) The series given by $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k + k \ln k}$

4. (12 points) Consider the series given by

$$\frac{b}{2} + \frac{3b^2}{4} + \frac{9b^3}{8} + \frac{27b^4}{16} + \cdots$$

   where b is a constant.

   (a) Write this series using summation notation.

   (b) For what values of b will the series converge?

   (c) Find the sum of the series when $b = 1/9$. Simplify your answer.

5. (15 points) Consider the series $\displaystyle\sum_{n=1}^{\infty} a_n = -8$ and let the partial sum $s_n = \displaystyle\sum_{i=1}^{n} a_i$. Which of the following statements are necessarily true? Write the entire word TRUE if the statement is always true. Write the entire word FALSE otherwise. Provide a short (1 or 2 sentences) explanation for each answer.

   (a) The sequence $\{a_n\}$ converges to -8.

   (b) $\lim_{n\to\infty} a_{n+1} = 0$.

(c) $\lim_{n\to\infty} s_n = -8$.

(d) If $s_3 = -7$ and $s_4 = -\frac{15}{2}$, then $a_4 = -\frac{1}{2}$.

(e) $\sum_{n=1}^{\infty} \left(a_n + \frac{1}{2}\right)$ converges.