1. (36 pts)

   Evaluate the following integrals and simplify your answers.

   (a) $\displaystyle\int_0^{\ln 3} (te^{-t} + e^{-t})\, dt$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\, dx$

   (c) $\displaystyle\int \frac{\sin(\theta)}{\cos^2(\theta) - 3\cos(\theta)}\, d\theta$

2. (24 pts) Determine whether the following integrals are convergent or divergent. Explain your reasoning fully for each integral. (If the integral converges, find its value, if you can. **If you use the Comparison Test, state this and evaluate the integral that you are using for comparison.**)

   (a) $\displaystyle\int_0^1 \frac{\sec^2(x)}{x\sqrt{x}}\, dx$

   (b) $\displaystyle\int_0^\infty \frac{e^x}{e^{2x}+1}\, dx$

3. (18 pts). For this problem, let $I = \displaystyle\int_0^1 \frac{1}{1+x^2}\, dx$.

   (a) Estimate I using the trapezoidal approximation T_2.

   (b) In this problem, $f(x) = \dfrac{1}{1+x^2}$, $f''(x) = \dfrac{2(3x^2-1)}{(1+x^2)^3}$ and $f^{(3)}(x) > 0$ on $(0,1)$. Use this information to find an error estimate for T_2. Briefly explain your reasoning.

   (c) How large do we have to choose n so that the approximation T_n to I is accurate to within 10^{-4}?

4. (22 pts) Consider the region $\mathcal{R}$, in the first quadrant, bounded by $y = 2x+1$, $y = 9-x^2$ and $x = 0$

   (a) On the graph below, sketch and shade the region $\mathcal{R}$. Be sure to label all intercepts and intersection points.

   (b) Set up, **but do not evaluate**, integrals to determine each of the following:

      i. The area of $\mathcal{R}$ using integration with respect to x.

      ii. The area of $\mathcal{R}$ using integration with respect to y.

      iii. The volume of the solid when $\mathcal{R}$ is rotated about the line $y = -1$.