
On the front of your bluebook, please write your name, lecture number, and instructor name. This exam is worth 150 points and has 8 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded.
  Please begin each problem on a new page.
- **Show all work and simplify your answers.** Name any theorem you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted except at the end of the test for **scanning and uploading your work to Gradescope**.

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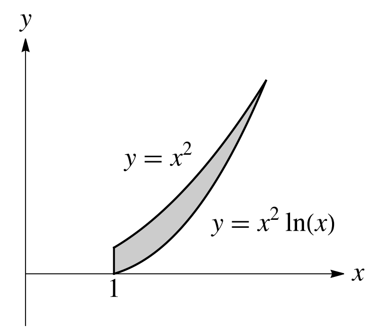
1. (22 pts) The shaded region $\mathcal{R}$, shown at right, is bounded above by $y = x^2$, below by $y = x^2 \ln x$, and on the left by $x = 1$.

   Set up integrals to find the following quantities. Simplify derivatives but otherwise do not evaluate the integrals.

   (a) Volume of the solid generated by rotating $\mathcal{R}$ about the line $x = 5$.

   (b) Volume of the solid generated by rotating $\mathcal{R}$ about the x-axis.

   (c) Area of the surface generated by rotating the lower border of $\mathcal{R}$ about the x-axis $\big($i.e., rotating the curve $y = x^2 \ln x\big)$.



2. (23 pts) Evaluate the integrals. Justify all indeterminate limits.

   (a) $\displaystyle\int \frac{dx}{(1+x^2)^{3/2}}$

   (b) i. $\displaystyle\int x^2 \ln x \, dx$ ii. $\displaystyle\int_0^1 x^2 \ln x \, dx$

3. (22 pts) Find the value the sequence or series converges to. If it does not converge, explain why not.

   (a) $\left\{ \dfrac{\sqrt{4n}}{1+\sqrt{n}} \right\}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n+2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^5\, 2^n}{5^n}$

4. (15 pts) Let $f(x) = x \ln x - x + 1$.

   (a) Use the formula for Taylor Series to find the polynomial $T_2(x)$ for $f(x)$ centered at $a = 1$.

   (b) Suppose $T_2(x)$ is used to approximate $f\left(\frac{3}{2}\right)$. By the Alternating Series Estimation Theorem, what is an error bound for the approximation? $\big($Note: The series corresponding to $f\left(\frac{3}{2}\right)$ is alternating and satisfies the conditions of the theorem.$\big)$

MORE PROBLEMS ON THE NEXT PAGE

5. (20 pts) Let $g(x) = \arctan\left(x^2\right)$.

   (a) Find a Maclaurin series for $g(x)$.

   (b) Use your answer for part (a) to find a Maclaurin series for $x^3 g'(x)$. Simplify your answer.

   (c) What is the sum of the series found in part (b)?

6. (14 pts) Consider the parametric curve $x = e^{t/2},\ y = 1 + e^{2t}$.

   (a) Find an equation of the line with slope 4 that is tangent to the curve.

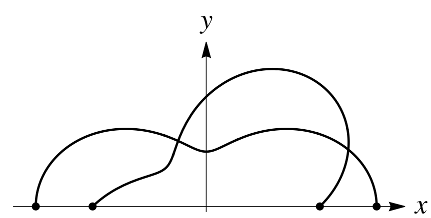
   (b) Eliminate the parameter to find a Cartesian equation of the curve. Simplify your answer.

7. (14 pts) Consider the curve $x^2 = 16(1 + y^2)$.

   (a) Find the vertices and asymptotes of the curve.

   (b) Find a polar representation $r = f(\theta)$ for the curve.

8. (20 pts) Consider the polar curves $r = 2 + \sin(2\theta)$ and $r = 2 + \cos(2\theta)$ in the 1st and 2nd quadrants, shown at right.



   (a) Find the (x, y) coordinates for the point that corresponds to $r = 2 + \sin(2\theta)$, $\theta = \frac{\pi}{6}$. Simplify your answer.

   (b) Set up (but <u>do not evaluate</u>) integrals to find the following quantities.

      i. Length of the curve $r = 2 + \sin(2\theta)$.

      ii. Area of the region inside $r = 2 + \sin(2\theta)$ and outside $r = 2 + \cos(2\theta)$. $\big($*Hint:* For the bounds, consider $\tan(2\theta)$.$\big)$