APPM 1360 **Exam 3** **Fall 2021**

On the front of your bluebook, please write your name, lecture number, and instructor name. This exam is worth 100 points and has 6 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded.
  Please begin each problem on a new page.
- **Show all work and simplify your answers.** Name any theorem you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted except at the end of the test for **scanning and uploading your work to Gradescope**.

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1. (30 pts) Determine whether the series is convergent or divergent. Be sure to fully justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{9n+4}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{(n-1)!}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(1+n)}{\ln(9+n^2)}$

2. (13 pts) The power series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} \frac{(x-10)^n}{2^{2n}}$ has a radius of convergence $R = 4$. For what values of x (if any) is the series conditionally convergent? absolutely convergent?

3. (14 pts)

   (a) Find a power series representation for $\dfrac{x}{1+x^3}$ centered at $a = 0$. Simplify your answer.

   (b) Use the power series to evaluate $\displaystyle\int_0^{0.9} \frac{x}{1+x^3}\, dx$. Express your answer in the form of a series.

4. (23 pts) Consider the function $f(x) = (2x+1)^{3/2}$.

   (a) Find the Taylor polynomial $T_1(x)$ for $f(x)$ centered at $a = 0$.

   (b) Use $T_1(x)$ to approximate the value of $f\left(\frac{1}{5}\right)$.

   (c) Use Taylor's Formula to find an error bound for the approximation found in part (b).

TURN OVER—More problems on the next page

5. (8 pts) Find the function $g(x)$ which has the power series representation

$$\sum_{n=1}^{\infty}(-1)^n \binom{1/3}{n} 9^n x^{n+1} \quad \text{for } |x| < \frac{1}{9}.$$

6. (12 pts) Consider the parametric curve $x = 2 + \sqrt{t},\ y = |t-1|,\ 0 \le t \le 4.$

   (a) Sketch the curve. Label the coordinates of the initial and terminal points. Indicate the direction of motion as the parameter increases.

   (b) Find a Cartesian representation of the curve.

Taylor Series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Taylor's Formula

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

Frequently Used Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \qquad R = 1$$