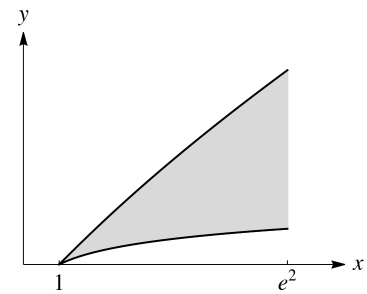

On the front of your bluebook, please write your name, lecture number, and instructor name. This exam is worth 100 points and has 6 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers.** Name any theorem you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted except at the end of the test for **scanning and uploading your work to Gradescope**.

---

1. (32 pts) The shaded region $\mathcal{R}_1$, shown at right, is bounded by $y = \sqrt{x}\ln x$, $y = \ln(\sqrt{x})$, and the line $x = e^2$ in the first quadrant. Set up (but do not evaluate) integrals to find the following quantities.

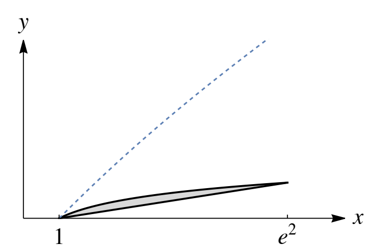


   (a) The volume of the solid obtained by rotating $\mathcal{R}_1$ about the line $x = -2$

   (b) The volume of the solid with $\mathcal{R}_1$ as the base and cross-sections perpendicular to the x-axis that are squares

   (c) The area of the surface generated by rotating the lower curve about the line $y = 6$

   Now connect the endpoints of the lower curve to form a line segment, $1 \le x \le e^2$. Consider the region $\mathcal{R}_2$, shown at right, bounded above by the lower curve and bounded below by the line segment. Set up an integral to find



   (d) The moment M_y of the region $\mathcal{R}_2$

2. (14 pts) Find the length of the curve $y = \sqrt{4 - x^2}$, $0 \le x \le \frac{1}{2}$, by evaluating an integral.

3. (14 pts) Solve the differential equation for y. Simplify your answer.

$$\frac{dy}{dx} = \frac{ye^x}{1 + e^x}$$

TURN OVER—More problems on the next page

4. (10 pts) Let $b_n = \dfrac{(n+2)!}{2n^2(n!)}$.

   (a) Does b_n converge? If so, what does it converge to?

   (b) Does $\displaystyle\sum_{n=1}^{\infty} b_n$ converge? If so, what does it converge to?

5. (14 pts) Consider the geometric series $\dfrac{2}{3} + \dfrac{2m}{9} + \dfrac{2m^2}{27} + \dfrac{2m^3}{81} + \cdots$.

   (a) For what values of m will the series converge?

   (b) Can the sum of the series equal $\frac{2}{5}$? If so, find the corresponding value of m.

6. (16 pts) Consider the series $\displaystyle\sum_{n=1}^{\infty} a_n$ with $a_n = \pi^{1/n} - \pi^{1/(n+1)}$. Let s_n represent the nth partial sum of the series.

   (a) Does a_n converge? If so, what does it converge to?

   (b) Find s_3. Simplify your answer.

   (c) Find an expression for s_n. Simplify your answer.

   (d) Does the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge? If so, what does it converge to?

Trigonometric identities

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(u/a) + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(u/a) + C$$

Center of Mass Integrals

$$M = \int_a^b \rho(f(x) - g(x))\,dx$$

$$M_y = \int_a^b \rho x(f(x) - g(x))\,dx$$

$$M_x = \int_a^b \frac{1}{2}\rho\left[(f(x))^2 - (g(x))^2\right]dx$$

$\bar{x} = \dfrac{M_y}{M}$ and $\bar{y} = \dfrac{M_x}{M}$