APPM 1360 **Exam 1** **Fall 2021**

On the front of your bluebook, please write your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers.** Name any theorem you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted except at the end of the test for **scanning and uploading your work to Gradescope**.

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1. (36 pts) Evaluate the integral.

   (a) $\displaystyle\int \left(\tan\theta + \frac{1}{\cos\theta}\right)^2 d\theta$ (b) $\displaystyle\int \frac{11}{(2x-1)(3x+4)}\,dx$ (c) $\displaystyle\int \frac{3x^3+18x-1}{x^2+6}\,dx$

2. (26 pts) Consider the integral $\displaystyle\int_0^{\pi/2} x\cos(2x)\,dx$.

   (a) Estimate the integral using the trapezoidal approximation T_3. Fully simplify your answer.

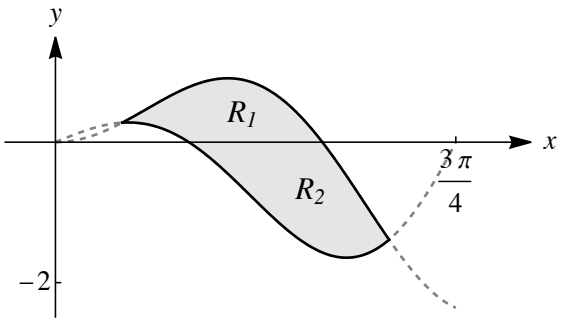
   (b) Find error estimate $|E_T|$ for the approximation T_3. You may leave your answer unsimplified. (*Hint:* The first derivative of $x\cos(2x)$ is $\cos(2x) - 2x\sin(2x)$.)

   (c) Find the exact value of the integral.

3. (16 pts) The shaded region shown below is bounded by $y = x\cos(2x)$ and $y = x\sin(2x)$. The region is composed of two smaller regions R_1 above the x-axis and R_2 below the x-axis. Set up (but do not evaluate) integrals to find the following quantities.

   (a) The area of shaded region R_1 which lies above the x-axis

   (b) The volume of the solid generated by rotating the entire shaded region (both R_1 and R_2) about the line $y = -2$



TURN OVER—More problems on the next page

4. (12 pts) Determine whether $\displaystyle\int_1^\infty \frac{e^{-x^3}}{\cosh(1)}\,dx$ is convergent or divergent. Justify your answer.

5. (10 pts) Let $f(x) = \dfrac{b-a}{(x-a)(x-b)}$ where a and b are constants, $0 < a < b$.

   Is $\displaystyle\int_{b+1}^\infty f(x)\,dx$ convergent or divergent? If convergent, find the value of the integral.

   If divergent, explain why. $\big($*Hint:* Let $g(x) = \ln|x-b| - \ln|x-a|$. Then $g'(x) = f(x).\big)$

END OF TEST

Trigonometric identities

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin^2(x) = \tfrac{1}{2}\left(1 - \cos(2x)\right)$$
$$\cos^2(x) = \tfrac{1}{2}\left(1 + \cos(2x)\right)$$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}(u/a) + C$$

Error Bounds for Trapezoidal and Midpoint Rules

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \qquad |E_M| \le \frac{K(b-a)^3}{24n^2}$$