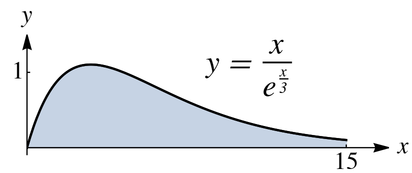
This exam is worth 150 points and has 6 questions.

Show all work and simplify your answers. Answers without proper justification will receive little to no credit unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic aids are not permitted.

1. (32 pts) Consider the shaded region bounded by $y = \dfrac{x}{e^{x/3}}$ and the x-axis, $0 \le x \le 15$, shown below. Note that $y' < 0$ for $x > 3$.



   (a) Set up (but do not evaluate) integrals to find the volume of the solid generated by rotating the region about the specified line:

      i. $x = -1$ ii. $y = 2$

   (b) Evaluate $\displaystyle\int \frac{x}{e^{x/3}}\,dx$.

   (c) Is the series $\displaystyle\sum_{n=3}^{\infty} n\sqrt[3]{e^{-n}}$ convergent or divergent?

2. (30 pts) The following problems are not related.

   (a) Evaluate $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-1}}$.

   (b) Does the sequence or series converge? If so, find the value it converges to. If not, explain why not.

      i. $\left\{\dfrac{n^2\cdot(2n-1)!}{(2n+1)!}\right\}$ ii. $\displaystyle\sum_{n=1}^{\infty} n\arctan\left(\tfrac{1}{n}\right)$ iii. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3n}\,7^{-n}}{4!}$

3. (28 pts) The function $\cosh(x)$ has the power series representation $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$, $R=\infty$.

   (a) Use $T_4(x)$ to approximate the value of $\cosh(2)$. Fully simplify your answer.

   (b) Use Taylor's Formula to find an error bound for the approximation in part (a). You may leave your answer in terms of cosh and/or sinh.

   (c) Find a power series representation for $\displaystyle\int x^8\cosh(x)\,dx$.

   (d) Find the sum of the series $\dfrac{2^2}{3^2\cdot 2!}+\dfrac{2^4}{3^4\cdot 4!}+\dfrac{2^6}{3^6\cdot 6!}+\dfrac{2^8}{3^8\cdot 8!}+\cdots$.

MORE PROBLEMS ON THE NEXT PAGE

4. (28 pts) The following three problems are not related.

   (a) Consider the parametric curve $x = 4\cos^2 t,\ y = 9\sin^2 t$. Find dy/dx and d^2y/dx^2 at $t = \frac{\pi}{4}$.

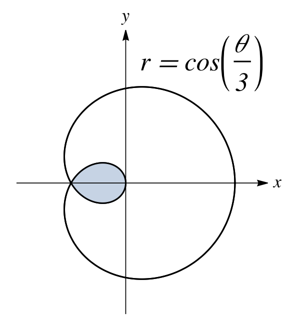
   (b) Find the length of the curve $x = e^t \cos t,\ y = e^t \sin t$, for $0 \le t \le \ln 5$. Fully simplify your answer.

   (c) Consider the parametric curve $x = \sqrt{t-1},\ y = \sqrt{t+8},\ t \ge 1$.

      i. Eliminate the parameter to find a Cartesian equation of the curve.

      ii. Identify the shape and sketch the curve. Label all intercepts.

5. (20 pts) Consider the polar curve $r = \cos(\theta/3)$, shown at right.



   (a) The curve has four intercepts, not including the pole. Find the (x, y) coordinates of the four intercepts.

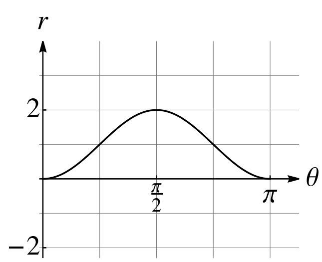
   (b) Set up (but do not evaluate) integrals to find the following quantities:

      i. the area of the inner loop of the curve

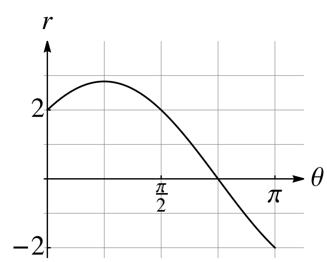
      ii. the length of the entire curve

6. (12 pts) Match each of the three r-θ graphs to its corresponding polar curve in the xy plane. Note that there are more polar curves than r-θ graphs. No justification is necessary for this problem.

(a)



(b)



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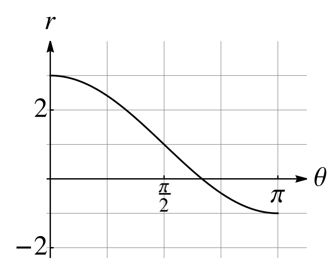
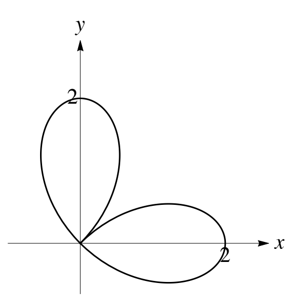


(I)



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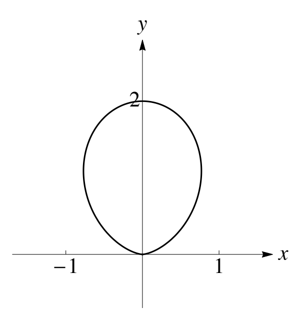

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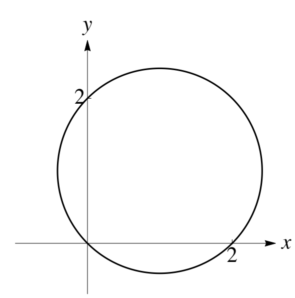

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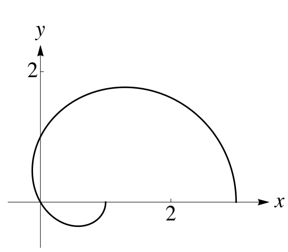
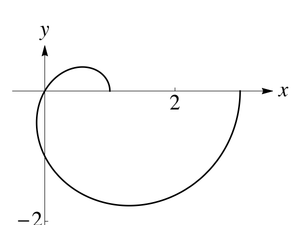


(V)



(VI)