APPM 1360 Exam 3 Fall 2020

This exam is worth 100 points and has 5 questions.

Show all work and simplify your answers. Answers without proper justification will receive little to no credit unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic aids are not permitted.

1. (27 pts) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Be sure to fully justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{6n-5}{n^3+n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\arctan(n))^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,[3\cdot 6\cdot 9\cdots(3n)]}{(n+1)!}$

2. (18 pts) Suppose the function $f(x)$ has a power series representation $\displaystyle\sum_{n=1}^{\infty} c_n(x-a)^n$ with an interval of convergence of $[-4,6)$.

   (a) Find the center and radius of convergence of the series.

   (b) Determine whether the following series are convergent or divergent, or if there is not enough information to determine the behavior of the series. Justify your answers.

   i. $\displaystyle\sum_{n=1}^{\infty} c_n 3^{2n}$ ii. $\displaystyle\sum_{n=1}^{\infty} \left(c_n + e^{-n}\right) 3^n$

   (c) Find a power series representation for $\displaystyle\int \frac{f(x)}{x-a}\,dx$. Express your answer in sigma notation in terms of c_n, using the value of a found previously.

3. (30 pts) Consider the function $g(x) = \ln\left(1+4x^2\right)$.

   (a) Find a power series representation for $g(x)$ centered at 0. Express your answer in sigma notation and simplify.

   (b) What is the radius of convergence of the series?

   (c) Use series to evaluate $\displaystyle\lim_{x\to 0} \frac{x^6}{\ln(1+4x^2)-4x^2+8x^4}$.

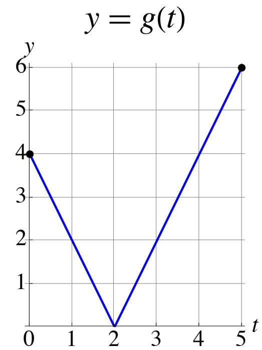
   (d) Use the Taylor polynomial $T_4(x)$ for $g(x)$ to approximate the value of $g\left(\frac{1}{2}\right)$. Fully simplify your answer.

   (e) Use the Alternating Series Estimation Theorem to determine the number of terms needed to estimate $g\left(\frac{1}{2}\right)$ with an error less than or equal to 10^{-2}. You may assume that the series satisfies the conditions of the theorem.

MORE PROBLEMS ON THE NEXT PAGE

4. (13 pts) Find the Taylor Series for $h(x) = 5/x^2$ centered at 1. Express your answer in sigma notation. Be sure to simplify your answer.

5. (12 pts) Consider the parametric curve $x = 1/e^t$, $y = g(t)$, $0 \le t \le 5$. The graph of $y = g(t)$ is shown below.



(a) Find the x and y coordinates of the initial and terminal points of the parametric curve.

(b) At which (x, y) point does the curve change direction?

(c) Find a Cartesian equation of the curve.

END OF TEST

Taylor Series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Taylor's Formula

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

Frequently Used Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \qquad R = 1$$