1. (28 pts) Consider the region $\mathcal{R}$ in the first quadrant bounded by $y = 2|x-2|$ and $y = (x-2)^2 + 1$.

   (a) Sketch the region $\mathcal{R}$. Label the x and y coordinates of the intersection points.

   (b) Set up but do not evaluate integrals to find the following quantities.

      i. The volume of the solid obtained by rotating $\mathcal{R}$ about $x = 3$.

      ii. The area of the surface generated by rotating the upper border of the region (the upper curve) about the line $y = 4$.

      iii. The area of the surface generated by rotating the lower border of the region (the lower curve) about the line $y = 4$.

2. (12 pts) Find the arc length of the curve $y = \dfrac{\ln x}{8} - x^2$ for $1 \le x \le 2$.

3. (10 pts) An 11-lb bag of sand is lifted at a constant rate. A hole in the bag causes the sand to leak out at a rate of 0.3 lb per second. Suppose it takes 10 seconds to lift the bag 7 ft off the ground.

   (a) Let $F(x)$ represent the weight of the sand at x ft above the ground, $0 \le x \le 7$. Find $F(x)$.

   (b) Set up (but do not evaluate) an integral to find the work done to lift the bag.

4. (12 pts) Solve the differential equation for y.

$$\frac{1}{2}\frac{dy}{dx} = \sqrt{y}\ln(x-1)$$

5. (12 pts) Let $b_m = \sqrt{m}\arctan\left(\frac{1}{\sqrt{m}}\right)$.

   (a) Does b_m converge? If so, what does it converge to?

   (b) Does $\displaystyle\sum_{m=1}^{\infty} b_m$ converge? If so, what does it converge to?

MORE PROBLEMS ON THE NEXT PAGE

6. (12 pts) Consider the series $\sum_{n=1}^{\infty}(c+2)^{-n}$.

   (a) Find all values of the constant c for which this series converges.

   (b) Suppose the sum of the series is $\frac{1}{20}$. Find c.

7. (14 pts) Consider the series $\sum_{n=1}^{\infty} a_n$ and its partial sums $s_n = \sum_{i=1}^{n} a_i$.

$$a_1 + a_2 + a_3 + \cdots = \square + 100 + \square + \cdots$$
$$\{s_1,\ s_2,\ s_3,\ \ldots\} = \{-20,\ \square,\ 200,\ \ldots\}$$

   (a) Find the missing values of a_1, s_2, and a_3.

   (b) Suppose that $s_{n+1} - s_n > 0$ and $s_n < 900$ for all $n \geq 1$.

      i. Does s_n converge?

      ii. Does a_n converge?

   Justify your answers. If there is not enough information provided, explain why.

END OF TEST

Trigonometric identities

$\sin(2x) = 2\sin(x)\cos(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(u/a) + C$$

Center of Mass Integrals

$$M = \int_a^b \rho(f(x) - g(x))\, dx$$
$$M_y = \int_a^b \rho x(f(x) - g(x))\, dx$$
$$M_x = \int_a^b \frac{1}{2}\rho\left[(f(x))^2 - (g(x))^2\right] dx$$
$$\bar{x} = \frac{M_y}{M} \text{ and } \bar{y} = \frac{M_x}{M}$$