This exam is worth 100 points and has 4 questions.

- On the front of your bluebook, write your name, a grading key, and APPM 1360.
- Show all work! Answers with no justification will receive no points.
- Begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

Some Trigonometric identities

\[ 2 \cos^2(x) = 1 + \cos(2x) \]
\[ 2 \sin^2(x) = 1 - \cos(2x) \]
\[ \sin(2x) = 2 \sin(x) \cos(x) \]
\[ \cos(2x) = \cos^2(x) - \sin^2(x) \]

Inverse Trigonometric Integral Identities

\[ \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C, \quad u^2 < a^2 \]
\[ \int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1}(u/a) + C \]
\[ \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}|u/a| + C, \quad u^2 > a^2 \]

Midpoint Rule

\[ \int_a^b f(x)dx \approx \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \text{ where } \Delta x = \frac{b - a}{n} \text{ and } \bar{x}_i = \frac{x_{i-1} + x_i}{2} \text{ and} \]
\[ |E_M| \leq \frac{K(b - a)^3}{24n^2}. \]

Trapezoidal Rule

\[ \int_a^b f(x)dx \approx \frac{\Delta x}{2} \left[ f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \text{ where } \Delta x = \frac{b - a}{n} \text{ and } |E_T| \leq \frac{K(b - a)^3}{12n^2}. \]
1. (30 points) Evaluate the following integrals:
   (a) $\int (x + 1)e^{-x}dx$
   (b) $\int \frac{x^3 + 6x}{(x^3 + 1)}dx$
   (c) $\int x^3\sqrt{1 - x^2}dx$

2. (25 points)
   (a) You have been tasked to approximate the integral of $f(x) = \cos(x)e^{-x}$ on the interval $[0, 3]$, using the midpoint rule. Find the number of subintervals needed so that the error of the approximation does not exceed $10^{-4}$. Simplify your final answer. Note: $f'(x) = -\sin(x)e^{-x} - \cos(x)e^{-x}$, $f''(x) = -\cos(x)e^{-x} + \sin(x)e^{-x} + \sin(x)e^{-x} + \cos(x)e^{-x} = 2\sin(x)e^{-x}$.

(b) Find the area bounded above by $x^2 + y^2 = 4$, bounded below by $x + y = 2$, and bounded on the right by $x = 1$.

3. (25 points)
   (a) Suppose $f(x)$ and $g(x)$ are continuous for all real numbers $x$ in $(-\infty, \infty)$ and $\int_{-\infty}^{\infty} f(x)dx$ is convergent, $\int_{-100}^{1} f(x)dx$ is divergent, and $\int_{-\infty}^{\infty} g(x)dx$ is divergent. For each integral below, determine if it must be convergent, divergent, or it is indeterminate (you cannot tell). No justification is necessary.
   i. $\int_{-\infty}^{\infty} f(x)dx$
   ii. $\int_{-100}^{1} f(x)dx$
   iii. $\int_{-\infty}^{1} (f(x) + g(x))dx$
   iv. $\int_{1}^{\infty} (f(x) + 1)dx$
   v. $\int_{-100}^{1} f(x)dx$
   vi. $\int_{-\infty}^{1} f(x)g(x)dx$

(b) Determine whether the following integral converges or diverges: $\int_{1}^{\infty} \frac{x + 1}{\sqrt{x^3 + 1}}dx$

4. (20 points)
   (a) Determine the form of the partial fraction decomposition of the following expressions. (You do not need to evaluate the coefficients!)
   i. $\frac{1}{(x - 3)^3(x + 2)}$
   ii. $\frac{1}{(x^2 + 3x + 5)^2}$
   iii. $\frac{1}{(x - 3)(x - 2)(x^2 + 4)}$

(b) Calculate $\int x^2 \ln(x) dx$ for $x > 0$. Hint: $\int \ln(x) dx = x \ln(x) - x + C$ for $x > 0$