1. [15 pts] The finite region bounded by the curves $y = \sqrt{x}$, $y = 1$ is revolved around the line $x = -\frac{1}{2}$.
   
   (a) [6 pts] Sketch the region with appropriate labels.
   
   (b) [9 pts] Using the method of cylindrical shells, find the volume of the resulting solid.

2. [20 pts] Consider the function $y = \int_{1}^{x} \sqrt{t^4 - 1} \, dt$ on the interval $1 \leq x \leq 3$.
   
   (a) [10 pts] Find the length of the curve.
   
   (b) [10 pts] Find the area of the surface obtained by rotating the curve about the $y$–axis.

3. [30 pts] The following problems are unrelated.
   
   (a) [10 pts] Solve the initial value problem: $\sqrt{xy} \frac{dy}{dx} = 1$, $y(1) = 1$. Write your answer in the form $y = f(x)$.
   
   (b) [20 pts] A thin flat plate of uniform density $\rho$ occupies the region bounded by the curves $y = x^2$ and $y = |x|$.
      
      i. [6 pts] Sketch the region, labeling it appropriately.
      
      ii. [2 pts] Using only your sketch, find $\bar{x}$, the $x$–coordinate of the centroid/center of mass of the region.
      
      iii. [12 pts] Find $\bar{y}$, the $y$–coordinate of the centroid/center of mass of the region.

4. [15 pts] Let $a_n = \ln \left(1 + \frac{1}{n}\right)$.
   
   (a) [5 pts] Does $\{a_n\}$ converge? If so, what is its limit?
   
   (b) [5 pts] Consider $\sum_{n=1}^{\infty} a_n$. Find an expression for the $n^{th}$ partial sum, $s_n$, simplifying your answer completely.
   
   (c) [5 pts] Using your answer to part (b), does $\sum_{n=1}^{\infty} a_n$ converge or diverge?
5. [20 pts] The following problems are unrelated.

(a) [5 pts] Find the sum of $2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$

(b) [12 pts] Do the following converge or diverge? Justify your answer.

   i. [4 pts] $\sum_{n=1}^{\infty} \left[ \frac{(-1)^n 4^{1-n}}{2n-1} + \frac{3}{\pi n/2} \right]$

   ii. [4 pts] $\sum_{n=1}^{\infty} \cos \frac{1}{n}$

   iii. [4 pts] The sequence $\{a_n\}$, where $a_n = f(n)$ for $n = 1, 2, 3, \ldots$, and $f(x)$ is a function with $|f(x)| \leq 100$ and $f'(x) > 0$ for $x > 0$.

(c) [3 pts] Can the Integral Test be used to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{|\sin n|}{2n-1}$?

   Answer either yes or no, providing a single-sentence justification of your answer.

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**Center of Mass Integrals**

\[
\begin{align*}
M &= \int_a^b \rho [f(x) - g(x)] \, dx \\
M_y &= \int_a^b \rho x [f(x) - g(x)] \, dx \\
M_x &= \int_a^b \frac{1}{2} \rho \left\{ [f(x)]^2 - [g(x)]^2 \right\} \, dx \\
\bar{x} &= \frac{M_y}{M} \quad \text{and} \quad \bar{y} = \frac{M_x}{M}
\end{align*}
\]