## **Instructions:**

- Write your name at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Be sure that your work is legible and organized.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.
- Turn in your hardcopy exam before you leave the room.

## **Summation Formulas**

• 
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
 •  $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$  •  $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$ 

## Half / Double Angle Formulas

• 
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
 •  $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 1 - 2\sin^2(\theta) \\ 1 + 2\cos^2(\theta) \end{cases}$  •  $\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$ 

**Inverse trig:** 

• 
$$\int \frac{1}{1+x^2} dx = \arctan(x) + C$$
 • 
$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$$

1. (22 pts) Evaluate the following:

(a) 
$$\int_{\pi}^{4\pi/3} \sin(x) e^{\cos(x)} dx$$
  
(b) 
$$\int \frac{\cosh(x)}{\sqrt{4 - \sinh^2(x)}} dx$$
  
(c) 
$$\frac{dy}{dx} \text{ if } y = x^{\sqrt{x-1}}$$

- 2. (26 pts) The following problems are not related.
  - (a) Find the absolute maximum and minimum values of  $h(x) = e^{\cos(x)}$  on the interval  $[\pi/2, 4\pi/3]$ .

3. (14 pts) A contractor is designing a cylindrical grain silo with a flat roof. The walls will cost \$4 per square foot to construct and the roof will cost \$100 per square foot. There is no floor. The silo must be able to hold 200 cubic feet of grain. What are the dimensions (radius and height) of the cylinder which minimize the cost of construction? You do not need to simplify your answer. Remember to include the correct units.

*Hint*: The surface area of the walls is  $S_{\text{wall}} = 2\pi rh$ , and the surface area of the roof is  $S_{\text{roof}} = \pi r^2$ . The total volume of the cylindrical silo is  $V = \pi r^2 h$ .

- 4. (24 pts) The following problems are not related.
  - (a) Evaluate  $\lim_{x \to 2} \frac{\sin(x-2) + 2x 4}{3x 6}$ .

(b) Evaluate 
$$\lim_{x \to 0} (\sin(x) + 1)^{\cot(x)}$$
.

- (c) Simplify  $\tan\left(\arcsin\left(\sqrt{1-4x^4}\right)\right)$ .
- 5. (14 pts) Potassium-40 has a half-life of 1.25 billion years. A sample of rock is measured to have 80% of the original mass of potassium-40 remaining. How old is the rock? Leave your answer in terms of logarithms and in units of billions of years.