Name: ______________________________

APPM 1350
Summer 2024

Exam 3

July 12

Instructions:

- Write your name at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Be sure that your work is legible and organized.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.
- Turn in your hardcopy exam before you leave the room.

Summation Formulas

- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\displaystyle\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Half / Double Angle Formulas

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 1 - 2\sin^2(\theta) \\ 1 + 2\cos^2(\theta) \end{cases}$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$
- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1-\cos(\theta)\right)}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1+\cos(\theta)\right)}$
- $\tan\left(\frac{\theta}{2}\right) = \begin{cases} \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} \\ \frac{\sin(\theta)}{1+\cos(\theta)} \\ \frac{1-\cos(\theta)}{\sin(\theta)} \end{cases}$

1. (28 pts) Evaluate the following using any technique.

   (a) $\displaystyle\int_1^4 \sqrt{x} + \frac{3}{\sqrt{x}}\,\mathrm{d}x$

   (b) $\displaystyle\int_{-4}^4 \sin(x) + \sqrt{16 - x^2}\,\mathrm{d}x$

   (c) $\displaystyle\int \sin\theta\sqrt{\cos\theta + 1}\,\mathrm{d}\theta$

   (d) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(\frac{2i}{n}\right)^2$

2. (24 pts) Consider the function $f(x) = x^2 - 9$ on the interval $[1, 4]$.

   (a) Find R_3, the right-endpoint Riemann approximation of the area under the curve using $n = 3$ subintervals.

   (b) Evaluate $\displaystyle\int_1^4 f(x)\,\mathrm{d}x$ using the Fundamental Theorem of Calculus (FTOC) part 2.

   (c) Write $\displaystyle\int_1^4 f(x)\,\mathrm{d}x$ as the limit of a Riemann sum with n equally spaced subintervals. You do not need to evaluate it.

3. (16 pts) Let $g(x) = \displaystyle\int_{\pi/3}^{\sqrt{x}} t\cos(t)\,\mathrm{d}t + 1$.

   (a) Find $g'(x)$.

   (b) Find the equation of the tangent line to g at $x = \pi^2/9$.

4. (14 pts) Suppose that $h''(x) = \sin(2x)$, $h'(\pi) = -1$, and $h(0) = 3$. Find $h(x)$.

5. (18 pts) Let $f(x) = x - \cos(2x)$.

   (a) Show that f has at least one root in the interval $[0, \pi/2]$.

   (b) Let $x_0 = \pi/4$. Use one iteration of Newton's method to approximate the root (that is, find x_1).

   (c) Show that for $x_0 = 7\pi/12$, Newton's method will fail.

$\boxed{\text{THIS IS THE END OF THE EXAM}}$

Scratch work

Be sure to label your problems.