Exam 2

## **Instructions:**

**APPM 1350** 

**Summer 2024** 

- Write your name at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Be sure that your work is legible and organized.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.
- Turn in your hardcopy exam before you leave the room.

## Half / Double Angle Formulas

• 
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
 •  $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 1 - 2\sin^2(\theta) \\ 1 + 2\cos^2(\theta) \end{cases}$  •  $\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$ 

• 
$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1-\cos(\theta)\right)}$$
 •  $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1+\cos(\theta)\right)}$  •  $\tan\left(\frac{\theta}{2}\right) = \begin{cases} \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}\\ \frac{\sin(\theta)}{1+\cos(\theta)}\\ \frac{1-\cos(\theta)}{\sin(\theta)} \end{cases}$ 

## **Angle Sum / Difference Formulas**

•  $\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \sin(\beta) \cos(\alpha)$ 

• 
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

• 
$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}$$

- 1. (40 pts) The following problems are not related.
  - (a) Find  $\frac{dy}{dx}$  if  $x \cos(y) + y^3 = \pi^{3/2}$ . You should solve for  $\frac{dy}{dx}$ , but you do not need to simplify your answer further.
  - (b) Use linearization to estimate  $(8.01)^{2/3}$ . You do not need to simplify your answer.
  - (c) Show that  $f(x) = x^2 3x 5$  satisfies the three hypotheses of Rolle's Theorem on the interval [-2, 5]. For what value(s) of c is the conclusion satisfied?

(d) Find the absolute maximum and minimum values of  $f(x) = \frac{x^2}{x+2}$  on the interval [-6, -5/2].

2. (16 pts) A small ship carrying nothing but rubber ducks hits an iceberg and begins to spill its cargo into the ocean. The rubber-duck spill is circular and the radius increases at a rate of 0.5m/s. At what rate is the area of the spill increasing when the radius is 10m?

3. (44 pts) Let 
$$f(x) = \frac{x^2 - 9}{x + 5}$$
. The first and second derivatives of  $f$  are:

$$f'(x) = \frac{x^2 + 10x + 9}{(x+5)^2}$$
$$f''(x) = \frac{32}{(x+5)^3}$$

- (a) Find the domain of f.
- (b) Find all x- and y-intercepts of f.
- (c) Find all vertical, horizontal, and slant asymptotes of f.
- (d) Find the intervals over which f is increasing and decreasing.
- (e) Find and classify all local extrema.
- (f) Find the intervals over which f is concave up and concave down. List points of inflection, if any.
- (g) Sketch f. Be sure to label all relevant quantities. You may use the blank graph provided on the next page if you wish.



## THIS IS THE END OF THE EXAM