Name: _____

APPM 1350
Summer 2024

Exam 1

June 14

Instructions:

- Write your name at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Be sure that your work is legible and organized.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.
- Turn in your hardcopy exam before you leave the room.

Half / Double Angle Formulas

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 1 - 2\sin^2(\theta) \\ 1 + 2\cos^2(\theta) \end{cases}$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$

- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1 - \cos(\theta)\right)}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1 + \cos(\theta)\right)}$
- $\tan\left(\frac{\theta}{2}\right) = \begin{cases} \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} \\ \frac{\sin(\theta)}{1+\cos(\theta)} \\ \frac{1-\cos(\theta)}{\sin(\theta)} \end{cases}$

Angle Sum / Difference Formulas

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

1. (22 pts) Evaluate the following limits or show that they do not exist. You may **not** use L'Hospital's Rule.

   (a) $\lim_{x\to 0} \dfrac{\tan(3x)}{2x}$

   (b) $\lim_{x\to 5} \dfrac{|x-5|}{x^2-4x-5}$

   (c) $\lim_{x\to\infty} \dfrac{1}{x^2+1}\sin(x)$

2. (26 pts) Find $f'(x)$ for the following functions. You do not need to simplify your final answers.

   (a) $f(x) = \sqrt{x+4}$, using the definition of the derivative.

   (b) $f(x) = \pi^2 + 1 + \dfrac{2}{x^5} + \cos(\pi x) + 2\sqrt[3]{x}$, using any method.

   (c) $f(x) = \dfrac{\sin(x^3+5)}{\cos(2x)}$, using any method.

3. (22 pts) Consider the function $f(x) = 2x^2\cos(x) + 1$.

   (a) Is f even, odd, or neither? Justify your answer.

   (b) Show that $f(x)$ has at least one root on the interval $[0, \pi]$.

   (c) Find the equation of the tangent line to f at $x = \pi/2$. You may leave your answer in point-slope form if you wish.

4. (14 pts) Consider the function

$$f(x) = \begin{cases} (x-3)^2 & x < 5 \\ ax + b & x = 5 \\ \dfrac{4a\sin(x-5)}{(x-5)} & x > 5 \end{cases}$$

   where a and b are unknown constants. Find the values of a and b such that f is continuous for all x. Use the definition of continuity to justify your answer.

5. (16 pts) Consider the function

$$f(x) = \frac{2x^2 + x - 3}{x^2 + x - 2}.$$

   Find all asymptotes and removable discontinuities of f. Be sure to justify your answers using limits.

THIS IS THE END OF THE EXAM

Scratch work

Be sure to label your problems.