Name: ____________________

APPM 1350
Summer 2023

Exam 2

June 30

Instructions:

- Write your name at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Be sure that your work is legible and organized.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.
- Turn in your hardcopy exam before you leave the room.

Half / Double Angle Formulas

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 1 - 2\sin^2(\theta) \\ 1 + 2\cos^2(\theta) \end{cases}$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$
- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1 - \cos(\theta)\right)}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1 + \cos(\theta)\right)}$
- $\tan\left(\frac{\theta}{2}\right) = \begin{cases} \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} \\ \frac{\sin(\theta)}{1+\cos(\theta)} \\ \frac{1-\cos(\theta)}{\sin(\theta)} \end{cases}$

Angle Sum / Difference Formulas

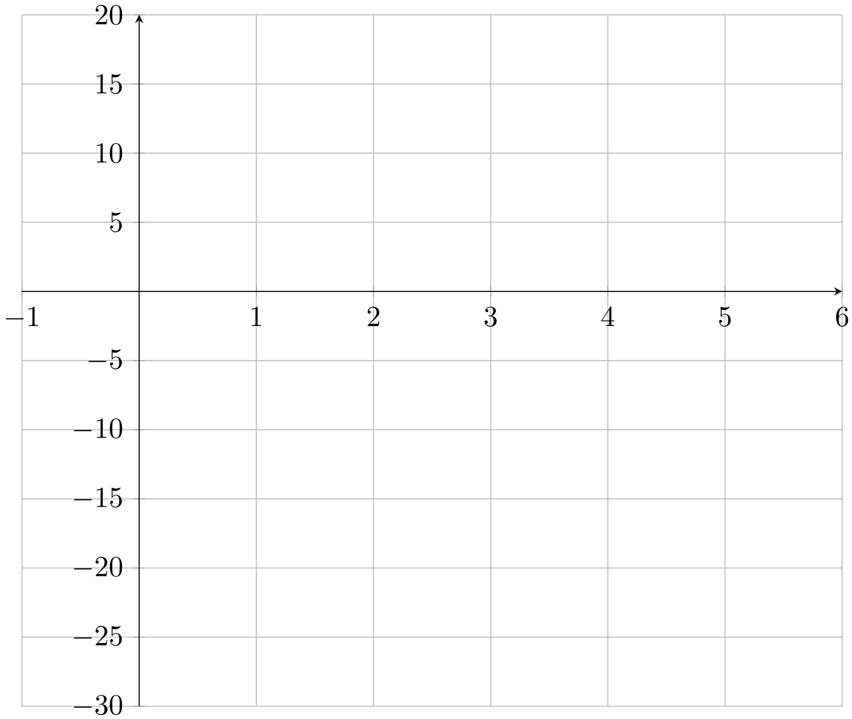
- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

1. (18 pts) Given the curve
$$y\sin^2(x) = \cos(x) + y^3$$
   (a) Find the derivative $\frac{dy}{dx}$ in terms of x and y.
   (b) Find the slope of the tangent line to the curve at $(\frac{\pi}{2}, 1)$.

2. (24 pts) Let $g(x) = \sqrt{25 - x}$
   (a) Find the linearization of $g(x)$ for $a = 9$ and use it to approximate $\sqrt{15.6}$. Your answer should be in decimal form.
   (b) Find the value(s) c that satisfy the conclusion of the Mean Value Theorem for $g(x)$ on $[0, 9]$. Leave your answer as an improper fraction.

3. (22 pts) Let x and y be two positive numbers under the constraint that $x + 2y = 4$. Determine the values of x and y that would maximize $(x + 1)(y + 2)$. Verify that it is a maximum by using either the first derivative test or the second derivative test.

4. (36 pts) Consider the function $y = x(x - 4)^3$,
its simplified derivative $y' = 4(x - 4)^2(x - 1)$,
and its simplified second derivative $y'' = 12(x - 4)(x - 2)$.
Justify your answers for each of the following problems.
   (a) Find the $x-$ and $y-$intercepts.
   (b) On what intervals is the function increasing? On what intervals is it decreasing?
   (c) Find the (x, y) coordinates of any local maximum and local minimum values, if they exist. If none exist, state this.
   (d) On what intervals is the function concave up? On what intervals is it concave down?
   (e) Find the (x, y) coordinates of any inflection points, if they exist. If none exist, state this.
   (f) Use the empty plot located on the next page to sketch the graph of this function. Carefully label all key features such as any intercepts, maximum(s), minimum(s), and inflection point(s). (Hint: there are no asymptotes for this function).



THIS IS THE END OF THE EXAM