Name: ____________________________

APPM 1350
Summer 2023

Exam 1

June 16

Instructions:

- Write your name at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Be sure that your work is legible and organized.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.
- Turn in your hardcopy exam before you leave the room.

Half / Double Angle Formulas

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 1 - 2\sin^2(\theta) \\ 1 + 2\cos^2(\theta) \end{cases}$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$
- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1 - \cos(\theta)\right)}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}\left(1 + \cos(\theta)\right)}$
- $\tan\left(\frac{\theta}{2}\right) = \begin{cases} \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} \\ \frac{\sin(\theta)}{1+\cos(\theta)} \\ \frac{1-\cos(\theta)}{\sin(\theta)} \end{cases}$

Angle Sum / Difference Formulas

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

1. (16 pts) The following two problems are not related.

   (a) Suppose $\csc(\theta) = \dfrac{3}{2}$, where $\dfrac{\pi}{2} \leq \theta \leq \pi$. Find the value of $\cot(\theta)$.

   (b) Find all values of x in the interval $[0, 3\pi]$ that satisfy $\cos^2(x) = \cos(2x) + 1$.

2. (20 pts) Evaluate the following limits or show that they do not exist. You may **not** use L'Hospital's Rule.

   (a) $\displaystyle \lim_{x \to -4} \frac{4 - x^2}{x - 4}$

   (b) $\displaystyle \lim_{x \to 1} \left[(x^2 - 1) \cos\left(\frac{\pi}{x^2 - 1} \right) + 3 \right]$

   (c) $\displaystyle \lim_{x \to 1} \frac{\sin(x - 1)}{x^2 + 4x - 5}$

3. (16 pts) The following questions are not related.

   (a) Let $f(x) = \sqrt{x - 2}$ and $g(x) = \dfrac{5x^2 + 1}{x^2 - 5}$. Find $(g \circ f)(x)$ and its domain in interval notation.

   (b) Show that $\sin^2(t) = \dfrac{1}{t^3 + 2}$ has at least one solution in the interval $[0, \frac{\pi}{2}]$. (Hint: you do not need to find the solution, you just need to show that it exists).

4. (10 pts) Consider the function,

$$g(x) = \begin{cases} \dfrac{x^2 + 2x - 3}{x - 1} & x < -1, \\ c & x = -1, \\ b\cos(\pi x) & x > -1 \end{cases}$$

   Recall the definition of continuity. Are there any constants b and c that make $g(x)$ continuous at $x = -1$? If so, determine the values of b and c. If not, explain why they do not exist. (Hint: $\cos(\theta)$ is an even function).

5. (14 pts) Find the horizontal and vertical asymptotes of $y = \dfrac{2x^2 - 7x - 4}{8x - 2x^2}$. <u>Justify using limits</u>. You may **not** use Dominance of Powers.

6. (24 pts) Differentiate the following.

   (a) $y = \tan(\theta)\sec(\theta)$

   (b) $f(x) = 5\sqrt[5]{x} - \dfrac{2}{x^4}$

   (c) $g(x) = \sqrt{\dfrac{4 + \sqrt{x}}{\sin(\frac{x}{3})}}$. Do not simplify $g'(x)$.

THIS IS THE END OF THE EXAM