Name ______________________ **Section** __________

APPM 1350
Summer 2022

Exam 2

June 24

Instructions:

- Write your name and section number at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, go to the scanning section of the room and upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.
- Turn in your hardcopy exam before you leave the room.

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1. (16 pts) Compute the derivatives of the following functions:

   a) $f(x) = \sin(x^2)$

   b) $g(x) = \sqrt{\cos(x)}$

   c) $h(x) = \dfrac{\tan(3x)}{x-1}$

   d) $j(x) = (3x+7)^3$

   Solution:

   a) $f'(x) = 2x\cos(x^2)$

   b) $g'(x) = -\dfrac{1}{2}\cos^{-\frac{1}{2}}(x)\sin(x) = -\dfrac{\sin x}{2\sqrt{\cos x}}$

   c) $h'(x) = \dfrac{(x-1)\sec^2(3x)\cdot 3 - \tan(3x)(1)}{(x-1)^2} = \dfrac{3(x-1)\sec^2(3x) - \tan(3x)}{(x-1)^2}$

   d) $j'(x) = 9(3x+7)^2$

2. (18 pts) Given the curve

$$xy^2 - x^2y = 6$$

   (a) Find the derivative $\dfrac{dy}{dx}$ in terms of x and y.

   (b) Find the tangent line to the above curve at the point $(1, -2)$.

   Solution:

   (a)

$$\begin{aligned}
\frac{d}{dx}(xy^2 - x^2y) &= \frac{d}{dx}(6) \\
x\cdot 2y\frac{dy}{dx} + y^2 - (x^2\frac{dy}{dx} + 2xy) &= 0 \\
2xy\frac{dy}{dx} - x^2\frac{dy}{dx} &= 2xy - y^2 \\
\frac{dy}{dx}(2xy - x^2) &= 2xy - y^2 \\
\frac{dy}{dx} &= \frac{2xy - y^2}{2xy - x^2}
\end{aligned}$$

   (b) $m = \left.\dfrac{dy}{dx}\right|_{(1,-2)} = \dfrac{2(1)(-2) - (-2)^2}{2(1)(-2) - (1)^2} = \dfrac{8}{5}$

   Tangent Line:

$$\begin{aligned}
y + 2 &= \frac{8}{5}(x-1) \\
y &= \frac{8}{5}x - \frac{18}{5}
\end{aligned}$$

3. (16 pts) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the deck when it is 8 m from the dock?

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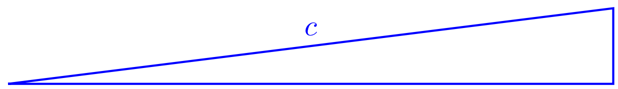
Solution:



x is the horizontal distance of the boat from the dock. c is the remaining length of the rope. Note that the height of the dock relative to the bow is constant. We are given that $\frac{dc}{dt} = 1$ m/s, and asked to find $\frac{dx}{dt}$. The equation we want to use to relate our variables is the Pythagorean theorem:

$$x^2 + 1 = c^2$$

Note that at the specified moment in time, $x = 8$ and $c = \sqrt{65}$. We plug in these values only after we've taken a derivative:

$$\begin{aligned} \frac{d}{dt}(x^2+1) &= \frac{d}{dt}(c^2) \\ 2x\frac{dx}{dt} &= 2c\frac{dc}{dt} \\ 2(8)\frac{dx}{dt} &= 2\sqrt{65}(1) \\ \frac{dx}{dt} &= \frac{\sqrt{65}}{8} \end{aligned}$$

4. (20 pts)

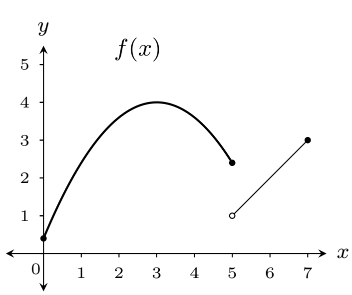
   (a) State the Mean Value Theorem.

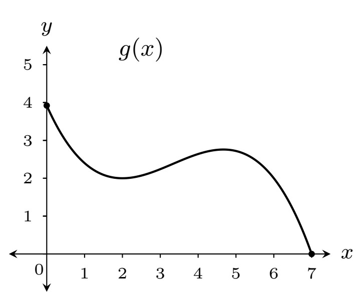
   (b) For each of the following functions $f(x), g(x)$, and $h(x)$. Answer the following:

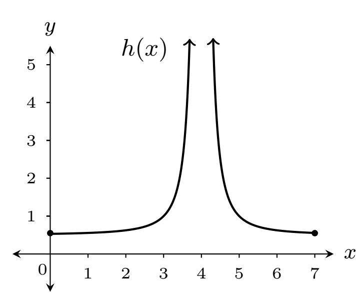
      i. Does the function satisfy the hypothesis of the Mean Value Theorem on [0,7]?

      ii. Does the function satisfy the conclusion?

      iii. Explain your answer from part (c). Approximate the values of c if the function satisfies the conclusion of the Mean Value Theorem is satisfied. If a function doesn't satisfy the conclusion of the Mean Value Theorem explain which condition in the hypothesis isn't satisfied.







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Solution:

(a) If a function f is

- continuous on an interval $[a, b]$ and
- differentiable on the interval (a, b),

then there exists a c, $a < c < b$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

(b) $f(x)$:

i. $f(x)$ does not satisfy the hypothesis since it is not continuous at $x = 5$.

ii. However it does still satisfy the conclusion.

iii. At the point $c \approx 2.5$ the slope of the tangent line will be the same as the slope of the secant line.

$g(x)$:

i. and ii. $g(x)$ satisfies the hypothesis of the mean value theorem and therefor it satisfies the conclusion.

iii. The values of c where the slope of the tangent line is the slope of the secant line are ≈ 1.5 and ≈ 5.5.

$h(x)$:

i. and ii. $h(x)$ does not satisfy the hypothesis of the mean value theorem nor does it satisfy the conclusion.

iii. The secant line is horizontal, and we can see that by the shape of the graph, because the function is discontinuous, there are no horizontal tangent lines.

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5. (30 pts) Given the following information for the function f:

- $f(x) = x^{\frac{2}{3}}(x+5)$
- $f'(x) = \dfrac{5(x+2)}{3x^{\frac{1}{3}}}$
- $f''(x) = \dfrac{10(x-1)}{9x^{\frac{4}{3}}}$

(a) Find all y- and x-intercepts and asymptotes.

(b) Find and classify all critical points.

(c) Determine the intervals of increasing and decreasing.

(d) Find any inflection points.

(e) Determine the intervals of concavity.

(f) Graph the function.

Solution:

(a) The y-intercept is 0. The x-intercepts are 0 and -5. There are no asymptotes.

(b) There is a local maximum at $x = -2$ and a local minimum at $x = 0$.

(c) The function is increasing on $(-\infty, -2) \cup (0, \infty)$. The function is decreasing on $(-2, 0)$.

(d) There is an inflection point at $x = 1$ and the second derivative is undefined at $x = 0$.

(e) The function is concave up from $(1, \infty)$. The function is concave down on $(-\infty, 0) \cup (0, 1)$.