Name _____   **Section** _____

Exam 2

Instructions:

- Write your name and section number at the top of each page.

- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.

- Name any theorem that you use and explain how it is used.

- Answers with no justification will receive no points unless the problem explicitly states otherwise.

- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.

- When you have completed the exam, go to the scanning section of the room and upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.

- Turn in your hardcopy exam before you leave the room.

1. (16 pts) Compute the derivatives of the following functions:

   a) $f(x) = \sin(x^2)$

   b) $g(x) = \sqrt{\cos(x)}$

   c) $h(x) = \dfrac{\tan(3x)}{x-1}$

   d) $j(x) = (3x+7)^3$

2. (18 pts) Given the curve
$$xy^2 - x^2 y = 6$$

   (a) Find the derivative $\dfrac{dy}{dx}$ in terms of x and y.

   (b) Find the tangent line to the above curve at the point $(1, -2)$.

3. (16 pts) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the deck when it is 8 m from the dock?


4. (20 pts)

   (a) State the Mean Value Theorem.

   (b) For each of the following functions $f(x), g(x)$, and $h(x)$. Answer the following:

      i. Does the function satisfy the hypothesis of the Mean Value Theorem on [0,7]?

      ii. Does the function satisfy the conclusion?

      iii. Explain your answer from part (c). Approximate the values of c if the function satisfies the conclusion of the Mean Value Theorem is satisfied. If a function doesn't satisfy the conclusion of the Mean Value Theorem explain which condition in the hypothesis isn't satisfied.
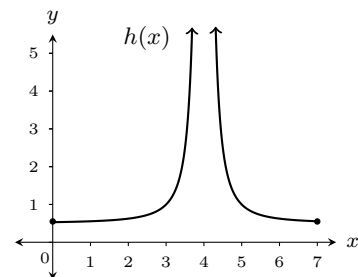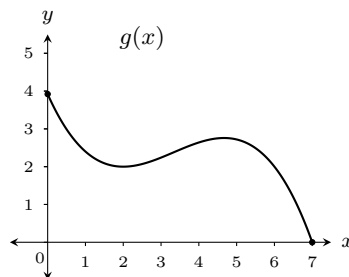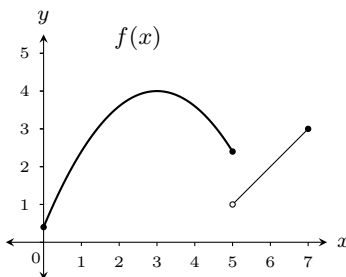
5. (30 pts) Given the following information for the function f:

- $f(x) = x^{\frac{2}{3}}(x + 5)$
- $f'(x) = \dfrac{5(x + 2)}{3x^{\frac{1}{3}}}$
- $f''(x) = \dfrac{10(x - 1)}{9x^{\frac{4}{3}}}$

(a) Find all y- and x-intercepts and asymptotes.

(b) Find and classify all critical points.

(c) Determine the intervals of increasing and decreasing.

(d) Find any inflection points.

(e) Determine the intervals of concavity.

(f) Graph the function.

$$f(x)$$